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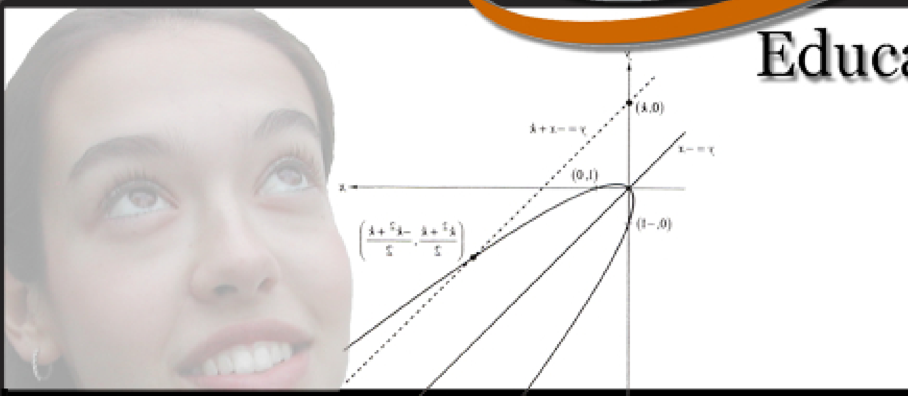

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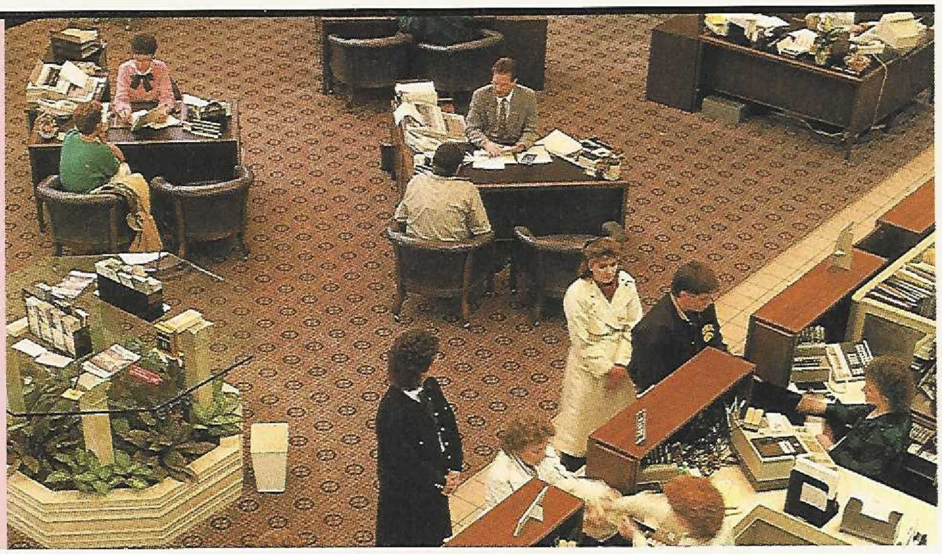
CHAPTER

2 Solving Equations and Inequalities

Bonnie has $3,000 invested at 8% simple interest per year. How much more money must she invest at 7% simple interest if she wants an income of $660 per year ($55 per month) from her investments?



2–1 ■ Algebraic notation and terminology

Algebraic terminology

In section 1–3, we defined a **variable** to be a symbol that represents an unspecified number. A variable is able to take on any one of the different values that it represents. In the relationship

$$y = 2x$$

y and x are variables since they both can assume various numerical values.

A constant is a symbol that does not change its value. In the relationship

$$y = 2x$$

2 is a constant. A number is a constant. If a symbol represents only one value, that symbol is a constant.

Any meaningful collection of variables, constants, grouping symbols, and signs of operations is called an algebraic expression. Examples of algebraic expressions would be

$$5xy,\quad \frac{xy}{z},\quad 2\ell + 2w,\quad \frac{x^2 - 1}{x^2 + 1},\quad 3x^2 + 2x - 1,\quad 5(a + 2b).$$

In an algebraic expression, terms are any constants, variables, or products or quotients of these. *Terms are separated by plus or minus signs.*

■ Example 2–1 A

Determine the number of terms in the algebraic expression.

The plus and minus signs separate the algebraic expression into three terms

1. $\underset{\text{1st}}{5x^2} + \underset{\text{2nd}}{2x} - \underset{\text{3rd}}{1}$ There are three terms

2. $\underset{\text{1st}}{x^2} + \underset{\text{2nd}}{y^2}$ There are two terms

3. $\underset{\text{1st}}{4x^5y^2z^4}$ There is one term

4. $\underset{\text{1st}}{a^2} + \underset{\text{2nd}}{\dfrac{b + c^2}{d}}$ There are two terms since the fraction bar forms a grouping. Observe that the second term has two terms in the numerator

▶ ***Quick check*** Determine the number of terms in $5 + x^2y - z$ and $4x^2 - \dfrac{2x + z}{y^2}$. ■

In the expression $5xy$, *each factor or grouping of factors is called the* **coefficient** *of the remaining factors.* That is, 5 is the coefficient of xy; x is the coefficient of $5y$; $5x$ is the coefficient of y; and so on. The 5 is called the **numerical coefficient,** and it tells us how many xy's we have in the expression.

Since we often talk about the numerical coefficients of a term, we will eliminate the word "numerical" and just say "coefficient." It will be understood that we are referring to the numerical coefficient. If no numerical coefficient appears in a term, the coefficient is *understood* to be 1.

■ Example 2–1 B

The algebraic expression $6x - 3y + z$ is thought of as the sum of terms $6x + (-3y) + z$, therefore 6 is the coefficient of x, -3 is the coefficient of y, and 1 is understood to be the coefficient of z.

▶ ***Quick check*** What are the coefficients in the algebraic expression $a^2 - 2a + 4b$? ■

A special kind of algebraic expression is a **polynomial.** The following are characteristics of a polynomial.

1. It has real number coefficients.
2. All variables in a polynomial are raised to only natural number powers.
3. The operations performed by the variables are limited to addition, subtraction, and multiplication.

A polynomial that contains just one term is called a **monomial;** a polynomial that contains two terms is called a **binomial;** and a polynomial that contains three terms is called a **trinomial.** Any polynomial that contains more than one term is

called a **multinomial,** but no special names are given to polynomials that contain more than three terms.

Example 2–1 C

Determine if each of the following algebraic expressions is a polynomial. If it is a polynomial, what name best describes it? If it is not a polynomial, state why it is not.

1. x, $4x$, 3, and $5x^2y$ are monomials.
2. $3x + 1$, $x + y$, and $81W^2 - 9T^2$ are binomials.
3. $5x^3 + 2y - 1$ and $z^2 + 9z - 10$ are trinomials.
4. $6x^3 - 2x^2 + 4x + 1$ is a polynomial of 4 terms.
5. $\dfrac{4}{x+2}$ is not a polynomial since it contains a variable in the denominator.

Note We should simplify any expression, before identifying it. Also, in an expression, the combining of all of the constant terms is understood to be a single term. For example, $x + 3 + \pi$ is thought of as $x + (3 + \pi)$ and is a binomial.

▶ ***Quick check*** Determine if each is a polynomial. If it is, what name best describes it? If it is not, state why it is not.

$5x^2y + 2z$; $\quad 5x^2y + \dfrac{2}{z}$ ■

Another way that we identify different types of polynomials is by the degree of the polynomial. *The* **degree** *of a polynomial in one variable is the greatest exponent of that variable in any one term.*

Example 2–1 D

Determine the degree of the polynomial.

1. $5x^3$ — Third degree because the exponent of x is 3
2. $x^4 - 2x^3 + 3x - 5$ — Fourth degree because the greatest exponent of x in any one term is 4

Note In example 2, the polynomial has been arranged in *descending powers* of the variable. This is the form that we will use when we write polynomials in one variable.

3. $4y^5 - 7y^2 + 3$ — Fifth degree because the greatest exponent of y in any one term is 5 ■

Algebraic notation

Many problems that we encounter will be stated verbally. These will need to be translated into algebraic expressions. While there is no standard procedure for changing a verbal phrase into an algebraic expression, the following guidelines should be of use.

1. Read the problem carefully, determining useful prior knowledge. Note what information is given and what information we are asked to find.
2. Let some letter represent one of the unknowns. Then express any other unknowns in terms of it.

3. Use the given conditions in the problem and the unknowns from step 2 to write an algebraic expression.

When translating verbal phrases into equations, we should be looking for phrases that involve the basic operations of addition, subtraction, multiplication, and division. Table 2–1 shows some examples of phrases that are commonly encountered. We will let x represent the unknown number.

Table 2–1

Phrase	Algebraic expression
Addition	
6 more than a number the sum of a number and 6 6 plus a number a number increased by 6 6 added to a number	$x + 6$
Subtraction	
6 less than a number a number diminished by 6 the difference of a number and 6 a number minus 6 a number less 6 a number decreased by 6 6 subtracted from a number a number reduced by 6	$x - 6$
Multiplication	
a number multiplied by 6 6 times a number the product of a number and 6	$6x$
Division	
a number divided by 6 the quotient of a number and 6 $\frac{1}{6}$ of a number	$\frac{x}{6}$

Example 2–1 E

Write an algebraic expression for each.

1. The product of a and b — $a \cdot b$
2. The sum of a and 4 — $a + 4$
3. x decreased by 9 — $x - 9$
4. y divided by 3 — $\frac{y}{3}$
5. A number increased by 6 — $n + 6$
6. Two times a number and that product decreased by 5 — $2n - 5$
7. A number divided by 3 and that quotient increased by 2 — $\frac{n}{3} + 2$

8. Twice the sum of x and 4 $\longrightarrow 2(x + 4)$

▶ ***Quick check*** Write an algebraic expression for the product of x and y. Write an algebraic expression for a number increased by 6.

Mastery points

Can you

- Identify terms in an expression?
- Identify a polynomial?
- Write an algebraic expression?

Exercise 2–1

Specify the number of terms in each expression. See example 2–1 A.

Examples $5 + x^2y - z$ $\qquad$ $4x^2 - \dfrac{2x + z}{y^2}$

Solutions 1st 2nd 3rd
Has three terms

1st 2nd
Has two terms

Terms are separated by plus and minus signs

1. $3x + 4y$
2. $5xyz$
3. $4x^2 + 3x - 1$
4. $x^3 - 4x + 7$
5. $\dfrac{6x}{5}$
6. $\dfrac{x}{3}$
7. $8xy + \dfrac{5y}{2} - 6x$
8. $\dfrac{15x^2 + y}{8}$
9. $5x^3 + (3x^2 - 4)$
10. $x^2 + a^2(y^2 - z)$
11. $(x + y + z)$
12. 7
13. $a^2(b + c) - x^2(y + z)$
14. $x^2 + \dfrac{y - z}{a} + c$

Determine the numerical coefficients of the following algebraic expressions. See example 2–1 B.

Example $a^2 - 2a + 4b$

Solution 1 is understood to be the coefficient of a^2, -2 is the coefficient of a, 4 is the coefficient of b.

15. $5x^2 + x - 4z$
16. $a^2b + 4ab^2 - ab$
17. $x - y - 3z$
18. $3x^4 - x^2 + x^2$
19. $-2a - b + c$

Determine if each of the following algebraic expressions is a polynomial. If it is a polynomial, what name best describes it? If it is not a polynomial, state why it is not. See example 2–1 C.

Examples $5x^2y + 2z$ $\qquad$ $5x^2y + \dfrac{2}{z}$

Solutions It is a polynomial. Since there are two terms, it is a binomial.

Not a polynomial because a variable is used as a divisor (appears in the denominator)

20. $ax^2 + bx + c$
21. $mx + b$
22. $5x^2 + 2x$
23. $y + \dfrac{1}{x}$
24. $\dfrac{a + b}{5} - c$
25. $\dfrac{a + b}{c} + d$
26. $4x^5 - 7x^3 + 3x - 2$
27. $9x^6 + 2x^2 + 4$

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Write an algebraic expression for each of the following. See example 2–1 E.

Examples	The product of x and y	A number increased by 6
Solutions	$x \cdot y$	Let x represent the number; hence $x + 6$

28. The sum of a and b
29. 3 times a, subtracted from b
30. 7 less than x
31. 5 more than y
32. The sum of x and y, divided by z
33. x times the sum of y and z
34. a decreased by 5
35. a decreased by b
36. $\frac{1}{2}$ of x, decreased by 2 times x
37. A number decreased by 12
38. A number added to 4
39. 3 times a number and that product increased by 1
40. A number divided by 5
41. 2 times the sum of a number and 4
42. A number decreased by 6 and that difference divided by 11

Review exercises

Perform the indicated operations. See section 1–8.

1. -5^2
2. $(-8)^2$
3. $10 - 6 \cdot 2$
4. $25 - 5 \cdot 2$
5. $100 \div 10 \cdot 2 + 2$
6. $28 - (8 - 12) - 3^2$

2–2 ■ Evaluating algebraic expressions

Substitution property

An extremely important process in algebra is that of calculating the numerical value of an expression when we are given specific replacement values for the variables. This process is called **evaluation.** To perform evaluation, we need the following **property of substitution.**

Property of substitution

If $a = b$, then a may be replaced by b or b may be replaced by a in any expression without altering the value of the expression.

Concept

When two things are equal, they can replace each other anywhere.

We frequently need to evaluate algebraic expressions. By using the substitution property and the order of operations, we can calculate the numerical value of an algebraic expression. For example, to find the distance (d) traveled when the rate (r) and time (t) are known, we use

$$d = rt$$

If the rate is 45 miles per hour and the time is 3 hours, we can substitute these values into the expression as follows:

$$d = (45)(3) = 135$$

The distance traveled is 135 miles. We replaced the respective variables representing rate and time with their values. We then carried out the indicated arithmetic.

Note When replacing variables with the numbers they represent, it is a good procedure to put each of the numbers inside parentheses.

■ Example 2–2 A

Evaluate the following expressions for the given real number replacement for the variable or variables.

1. $x^2 + 2x - 7$, when $x = 4$

The expression would be $(\quad)^2 + 2(\quad) - 7$ without the x. Substituting 4 for each x, we have $(4)^2 + 2(4) - 7$. Using the order of operations, we have

$= 16 + 2(4) - 7$	Exponents
$= 16 + 8 - 7$	Multiply
$= 24 - 7$	Add
$= 17$	Subtract

Therefore the expression $x^2 + 2x - 7$ evaluated for $x = 4$ is 17.

2. $5a - b + 2(c + d)$, when $a = 2$, $b = 3$, $c = -2$, and $d = -3$.

$5a - b + 2(c + d)$	Original expression
$= 5(\quad) - (\quad) + 2[(\quad) + (\quad)]$	Expression ready for substitution
$= 5(2) - (3) + 2[(-2) + (-3)]$	Substitute
$= 5(2) - (3) + 2[-5]$	Order of operations, groups
$= 10 - (3) + (-10)$	Multiply
$= 7 + (-10)$	Subtract
$= -3$	Add

3. $4ab - c^2 + 3d$, when $a = 2$, $b = 3$, $c = -2$, and $d = -3$.

$4ab - c^2 + 3d$	Original expression
$= 4(\quad)(\quad) - (\quad)^2 + 3(\quad)$	Expression ready for substitution
$= 4(2)(3) - (-2)^2 + 3(-3)$	Substitute
$= 4(2)(3) - (4) + 3(-3)$	Order of operations, exponents
$= 8(3) - 4 + 3(-3)$	Multiply
$= 24 - 4 + (-9)$	Multiply
$= 20 + (-9)$	Subtract
$= 11$	Add

▶ ***Quick check*** Evaluate $3a - 2(c - d) + b$ when $a = 2$, $b = 3$, $c = -2$, and $d = -3$ ■

To evaluate an algebraic expression

1. Write parentheses in place of each variable.
2. Place the value that the variable is representing inside the parentheses.
3. Perform the indicated operations according to the order of operations.

Formulas

A **formula** expresses a relationship between quantities in the physical world, for example, $d = rt$.

Example 2–2 B

Evaluate the following formulas for the real number replacements of the variables.

1. The volume (V) of a rectangular solid is found by multiplying length (ℓ) times width (w) times height (h). The formula then reads $V = \ell wh$. Find the volume in cubic feet if $\ell = 12$ feet, $w = 4$ feet, and $h = 5$ feet.

$V = \ell wh$	Original formula
$V = (\quad)(\quad)(\quad)$	Formula ready for substitution
$V = (12)(4)(5)$	Substitute
$V = (48)(5)$	Order of operations, multiply
$V = 240$	Multiply

The volume is 240 cubic feet.

2. If we know the temperature in degrees Fahrenheit (F), the temperature in degrees Celsius (C) can be found by the formula $C = \frac{5}{9}(F - 32)$. Find the temperature in degrees Celsius if the temperature is 86 degrees Fahrenheit.

$C = \frac{5}{9}(F - 32)$	Original formula
$C = \frac{5}{9}[(\quad) - 32]$	Formula ready for substitution
$C = \frac{5}{9}[(86) - 32]$	Substitute
$C = \frac{5}{9}(54)$	Order of operations, groups first
$C = 30$	Multiply

The temperature is 30 degrees Celsius.

3. The perimeter* (P) of a rectangle is found by the formula $P = 2\ell + 2w$, where ℓ is the length of the rectangle and w is the width. Find the perimeter of the rectangle in meters if $\ell = 8$ meters and $w = 5$ meters.

$P = 2\ell + 2w$	Original formula
$P = 2(\quad) + 2(\quad)$	Formula ready for substitution
$P = 2(8) + 2(5)$	Substitute
$P = 16 + 10$	Order of operations, multiply
$P = 26$	Add

The perimeter of the rectangle is 26 meters.

*The perimeter is the distance around a closed geometric figure.

4. A formula in electricity is $I = \dfrac{E}{R}$, where I represents the current measured in amperes in a certain part of a circuit, E represents the potential difference in voltage across that part of the circuit, and R represents the resistance in ohms in that part of the circuit. Find I in amperes if $E = 110$ volts and $R = 44$ ohms.

$I = \dfrac{E}{R}$ Original formula

$I = \dfrac{(\ )}{(\ )}$ Formula ready for substitution

$I = \dfrac{(110)}{(44)}$ Substitute

$I = \dfrac{5}{2} = 2\dfrac{1}{2}$ Reduce and change to a mixed number

The current is $2\frac{1}{2}$ amperes.

▶ ***Quick check*** Evaluate $I = \dfrac{E}{R}$ when $E = 220$ volts and $R = 11$ ohms. ■

Subscripts

In some formulas, two or more measurements of the same unit may be given. It is customary to label these by using **subscripts.** To illustrate, given two different measurements of pressure in a science experiment, we might label them

$$P_1 \text{ and } P_2$$

The 1 and 2 are the subscripts. Subscripts are always written to the lower right of the letter. The symbols above are read "P sub-one" and "P sub-two."

Note Do not confuse a subscript, such as P_2, that helps distinguish between different measurements, and an exponent, such as P^2, that indicates the number of times a given base is used as a factor in an indicated product. Subscripts are written to the lower right of the symbol. Exponents are written to the upper right of the symbol.

P_2 $\qquad$ P^2

Subscript $\qquad$ Exponent

■ ***Example 2–2 C***

1. In a science problem $\dfrac{V_1}{V_2} = \dfrac{T_1}{T_2}$, where V_1 and V_2 represent different measurements of volume and T_1 and T_2 represent different measurements of temperature.

2. In $\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$, R_1, R_2, and R_3 represent three different measurements of resistance and R_t represents the total resistance in the electrical circuit.

▶ ***Quick check*** Evaluate $C_t = \frac{C_1 \cdot C_2}{C_1 + C_2}$ when $C_1 = 4$ and $C_2 = 6$. ■

Problem solving

The following sets of word problems are designed to help us interpret word phrases and write expressions for them in algebraic symbols.

■ ***Example 2–2 D***

Write an algebraic expression for each of the following word phrases.

1. Nancy can type 90 words per minute. How many words can she type in n minutes?

   If Nancy can type 90 words in one minute, then we multiply

   $90 \cdot n$ or $90n$

   to obtain the number of words she can type in n minutes.

2. If John has n dollars in his savings account and on successive days he deposits \$15 and then withdraws \$34 to make a purchase, write an expression for the balance in his savings account.

   We *add* the deposits and *subtract* the withdrawals. Thus

   $n + 15 - 34$

   represents the balance in John's savings account after the two transactions.

3. A woman paid d dollars for a 30-pound bag of dog food. How much did the dog food cost her per pound?

   The price per pound is found by dividing the total cost by the number of pounds. Thus the price per pound of the dog food is represented by $\frac{d}{30}$ dollars.

▶ ***Quick check*** Express the cost in dollars of x cassette tapes if each tape costs \$2.95. ■

Mastery points

Can you

- Evaluate an algebraic expression?
- Evaluate a formula?
- Use subscripts?
- Write an algebraic expression?

Exercise 2–2

Evaluate the following expressions if $a = 2$, $b = 3$, $c = -2$, $d = -3$. See example 2–2 A.

Example $3a - 2(c - d) + b$

Solution

$$
\begin{aligned}
&= 3(\quad) - 2[(\quad) - (\quad)] + (\quad) && \text{Expression ready for substitution}\\
&= 3(2) - 2[(-2) - (-3)] + (3) && \text{Substitute}\\
&= 3(2) - 2(1) + 3 && \text{Order of operations, groups}\\
&= 6 - 2 + 3 && \text{Multiply}\\
&= 7 && \text{Subtract and add}
\end{aligned}
$$

1. $2a + b - c$
2. $(a + b)$
3. $3a - 2b - (c + d)$
4. $a - 3(c + b)$
5. $(3a + 2b)(a - c)$
6. $2ab(c + d)$
7. $ac - bd$
8. $3c - 2(3a + b)$
9. $7a - d(6b + c)$
10. $(3a - 5c)(2b - 4d)$
11. $(5c - 3a)(4d - 2b)$
12. $(5c - d)4a$
13. $5a + 7b - 3c(a - d)$
14. $(4a + b) - (3a - b)(c + 2d)$
15. $a^2 - c^2$
16. $b^2 + 2d^2$
17. $3ab - 4c^2 + d$
18. $(c + d)^2$
19. $(c - d)^2$
20. $a^2b^2 + c^2d^2$
21. $3ac - 2a^2c^2$
22. $ab - ac$
23. $(ab)^2 - (ac)^2$
24. $(c - d)^2(a + b)$
25. $a^3b - c^3d$
26. $c^2 - d^3$
27. $3d^2 - 2c^3$
28. $(3d - 5c)^3$

Evaluate the following formulas. See examples 2–2 B and C.

Examples $I = \dfrac{E}{R}$, $E = 220$ and $R = 11$ $\qquad C_t = \dfrac{C_1 \cdot C_2}{C_1 + C_2}$, $C_1 = 4$ and $C_2 = 6$

Solutions

$$
I = \frac{(\quad)}{(\quad)} = \frac{(220)}{(11)} = 20
$$

$$
\begin{aligned}
C_t &= \frac{(\quad)(\quad)}{(\quad) + (\quad)} && \text{Formulas ready for substitution}\\
&= \frac{(4)(6)}{(4) + (6)} && \text{Substitute}\\
&= \frac{24}{10} && \text{Order of operations}\\
&= \frac{12}{5} && \text{Reduce}
\end{aligned}
$$

29. $I = \dfrac{E}{R}$, $E = 220$ and $R = 33$
30. $V = \ell wh$, $\ell = 7$, $w = 5$, and $h = 6$
31. $I = prt$, $p = 1{,}000$; $r = 0.08$; and $t = 2$
32. $F = ma$, $m = 18$ and $a = 6$
33. $W = I^2R$, $I = 12$ and $R = 2$
34. $V = k + gt$, $k = 24$, $g = 9$, and $t = 4$
35. $A = \dfrac{1}{2}h(b_1 + b_2)$, $h = 6$, $b_1 = 8$, and $b_2 = 10$
36. $\ell = a + (n - 1)d$, $a = 2$, $n = 14$, and $d = 3$
37. $A = p + pr$, $p = 2{,}000$ and $r = 0.07$
38. $H = \dfrac{D^2N}{2}$, $D = 4$ and $N = 6$

$\boxed{39.}$ $A = \dfrac{I^2R - 120E^2}{R}$, $E = 5$, $I = 12$, and $R = 100$

40. $C_t = \dfrac{C_1 \cdot C_2}{C_1 + C_2}$, $C_1 = 6$ and $C_2 = 12$

41. $S = \dfrac{1}{2}gt^2$, $g = 32$ and $t = 4$

42. $R_t = \dfrac{R_1 \cdot R_2}{R_1 + R_2}$, $R_1 = 8$ and $R_2 = 12$

43. $V_1 = \dfrac{V_2P_2}{P_1}$, $V_2 = 18$, $P_2 = 12$, and $P_1 = 36$

44. $V_2 = \dfrac{V_1T_2}{T_1}$, $V_1 = 12$, $T_1 = 4$, and $T_2 = 14$

Evaluate the following formulas. See examples 2–2 B and C.

45. Find the horsepower (h) required by a hydraulic pump when it needs to pump 10 gallons per minute (g) and the pounds per square inch (p) equal 3,000. Use $h = \dfrac{g \cdot p}{1{,}714}$.

46. The required ratio of gearing (R) of a milling machine is given by $R = (A - N) \cdot \dfrac{40}{A}$, where N = required number of divisions and A = approximate number of divisions. Find R when $A = 280$ and $N = 271$.

$\boxed{47.}$ In a gear system, the velocity (V) of the driving gear is defined by $V = \dfrac{vn}{N}$, where v = velocity of follower gear, n = number of teeth of follower gear, and N = number of teeth of driving gear. Find V when $v = 90$ revolutions per minute, $n = 30$ teeth, $N = 65$ teeth.

48. The tap drill size (T) of a drill needed to drill threads in a nut is given by $T = D - \dfrac{1}{N}$, where D = diameter of the tap and N = number of threads per inch. Find T with a $\dfrac{1}{2}$-inch diameter tap and 13 threads per inch.

49. It is necessary to drag a box 600 feet across a level lot in 3 minutes. The force required to pull the box is 2,000 pounds. What is the horsepower (h) needed to do this if horsepower is defined by $h = \dfrac{\ell \cdot w}{33{,}000 \cdot t}$, where ℓ = length to be moved, w = force exerted through distance ℓ, and t = time in minutes required to move the box through ℓ?

50. A pulley 12 inches in diameter that is running at 320 revolutions per minute is connected by a belt to a pulley 9 inches in diameter. How many revolutions per minute will the smaller pulley make if $s = \dfrac{SD}{d}$, where s = speed of smaller pulley and d = diameter of smaller pulley, S = speed of larger pulley and D = diameter of larger pulley?

Write an algebraic expression for the following word statements. See example 2–2 D.

Example Express the cost in terms of dollars and in terms of cents for x cassette tapes if each tape costs \$2.95.

Solution If we are buying x tapes at \$2.95 each, then we must multiply x by \$2.95. The algebraic expression in terms of dollars would be $2.95 \cdot x$ and in terms of cents, it would be $295 \cdot x$.

51. Jim enters 85 keystrokes per minute on the computer. How many keystrokes can he enter in m minutes?

52. Express the cost in dollars of h gallons of heating oil if each gallon costs \$1.08.

53. A 10-pound box of candy costs y dollars. How much does the candy cost per pound?

54. Mike paid \$25 for a ticket to a play. If the play lasted h hours, what did it cost him per hour to see the play?

$\boxed{55.}$ Arlene has n nickels and d dimes in her purse. Express in cents the amount of money she has in her purse. (*Hint:* n nickels is represented by $5n$.)

56. Jack has q quarters, d dimes, and n nickels. Express in cents the amount of money Jack has.


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57. Susan is p years old now. Express her age (a) 12 years from now, (b) 5 years ago.

58. Ann is 3 years old. If Jan is n times as old as Ann, express Jan's age. Express-Jan's age 8 years ago.

59. Bill's savings account has a current balance of \$258. He makes a withdrawal of n dollars and then makes a deposit of m dollars. Express his new balance in terms of n and m.

60. Paula has a balance of n dollars in her checking account. She makes a deposit of \$36 and then writes 3 checks for m dollars each. Express her new balance in terms of n and m.

61. Pete has c cents, all in half-dollars. Write an expression for the number of half-dollars Pete has.

62. If x represents a whole number, write an expression for the next greater whole number.

63. If y represents an even integer, write an expression for the next greater even integer.

64. If z represents an odd integer, write an expression for the next greater odd integer.

65. If Larry is f feet and t inches tall, how tall is Larry in inches?

66. John earns \$1,000 more than twice what Terry earns in a year. If Terry earns d dollars, write an expression for John's annual salary.

67. Jean's annual salary is \$2,000 less than n times Lisa's salary. If Lisa earns \$25,000 per year, express Jean's annual salary.

68. Express the total cost of purchasing x cans of tuna at 69¢ per can on Friday and y cans of the same tuna at 57¢ per can on Saturday.

69. A gallon of primer paint costs \$9.95 and a gallon of latex-base paint costs \$12.99. Express the cost in dollars of p gallons of primer and q gallons of latex-base paint.

70. Paula enters x calculations per minute on the calculator and Leigh enters 7 calculations per minute less than Paula. Write an expression for the number of calculations Leigh enters in 35 minutes.

Review exercises

Perform the indicated operations. See sections 1–4 and 1–5.

1. $(-12) + 6$
2. $4 + (-8)$
3. $10 - 18$
4. $9 - (-9)$
5. $-6 - (-6)$
6. $(-14) + (-7)$

2–3 ■ Algebraic addition and subtraction

Since algebraic expressions (including polynomials) represent real numbers when the variables are replaced by real numbers, the ideas and properties that apply to operations with real numbers also apply to algebraic expressions.

The distributive property

The distributive property is the only property that establishes a relationship between addition (or subtraction) and multiplication in the same expression. The distributive property allows us to change certain multiplication problems into sums or differences.

Distributive property

For every real number a, b, and c,

$$a(b + c) = ab + ac \text{ and } a(b - c) = ab - ac$$

Concept

If a number is being used to multiply the sum or difference of two others, it is "distributed" to them both. That is, it multiplies them both.

Example 2–3 A

The following are applications of the distributive property.

Each term inside the parentheses is multiplied by 3

1. $3(4 + 5) = (3 \cdot 4) + (3 \cdot 5)$
$= 12 + 15$
$= 27$

Note Since we are able to add the numbers inside the parentheses, our solution without using the distributive property would be

$$3(4 + 5) = 3(9) = 27$$

2. $2(3 + a) = 2 \cdot 3 + 2 \cdot a$
$= 6 + 2a$

Note In this example, we could not add the numbers inside the parentheses. Therefore the multiplication could only be carried out by using the distributive property.

We are now going to use the distributive property to carry out addition and subtraction of algebraic expressions and to remove grouping symbols.

Like terms

We first need to define the types of quantities that can be added or subtracted. *We can add or subtract only like, or similar, quantities.* **Like terms or similar terms are terms whose variable factors are the same.**

Note For two or more terms to be called like terms, the variable factors of the terms, along with their respective exponents, must be identical. However the numerical coefficients of these identical variable factors may be different.

Example 2–3 B

1. $3a^2b^3$ and $-2a^2b^3$ are like terms because the variables are the same (a and b) and the respective exponents are the same (a is to the second power in each term and b is to the third power).
2. $2x^2y$ and $2xy^2$ both contain the same variables but are *not* like terms because the exponents of the respective variables are not the same.

▶ ***Quick check*** Are $4a^2b^2$ and $4a^3b^3$ like terms?

Addition and subtraction

Using the definition of like terms and the distributive property, we are ready to carry out addition and subtraction of algebraic expressions. Consider the following example:

$$3a + 4a$$

Using the distributive property, the expression can be written

$$3a + 4a = (3 + 4)a = 7a$$

Note The process of addition or subtraction is performed only with the numerical coefficients. *The variable factor and its exponent remain unchanged.*

Combining like terms

1. Identify the like terms.
2. If necessary, use the commutative and associative properties to group together the like terms.
3. Combine the numerical coefficients of the like terms and multiply that by the variable factor.
4. Remember that y is the same as $1 \cdot y$ and $-y$ is the same as $-1 \cdot y$.

■ *Example 2–3 C*

Perform the indicated addition and subtraction.

1.
$$\begin{aligned} &5x + 7x && \text{Identify like terms} \\ &\quad = (5 + 7)x && \text{Distributive property} \\ &\quad = 12x && \text{Add numerical coefficients} \end{aligned}$$

2.
$$\begin{aligned} &4ab + 3ab && \text{Identify like terms} \\ &\quad = (4 + 3)ab && \text{Distributive property} \\ &\quad = 7ab && \text{Add numerical coefficients} \end{aligned}$$

3.
$$\begin{aligned} &y + 3y - 2y && \text{Identify like terms} \\ &\quad = (1 + 3 - 2)y && \text{Distributive property} \\ &\quad = 2y && \text{Combine numerical coefficients} \end{aligned}$$

4.
$$\begin{aligned} &2x + 6y + 5x - 3y && \text{Identify like terms} \\ &\quad = 2x + 5x + 6y - 3y && \text{Commutative property} \\ &\quad = (2x + 5x) + (6y - 3y) && \text{Associative property} \\ &\quad = (2 + 5)x + (6 - 3)y && \text{Distributive property} \\ &\quad = 7x + 3y && \text{Combine numerical coefficients} \end{aligned}$$

Note Because of the commutative and associative properties, we can rearrange the expression and combine like terms.

5.
$$\begin{aligned} 6x^2 - 4x + 3x - 2x^2 &= (6x^2 - 2x^2) + (-4x + 3x) \\ &= (6 - 2)x^2 + (-4 + 3)x \\ &= 4x^2 - 1x = 4x^2 - x \end{aligned}$$

6.
$$\begin{aligned} 5x^2y^2 - 2xy^2 + 3x^2y^2 + 5xy^2 &= (5x^2y^2 + 3x^2y^2) + (-2xy^2 + 5xy^2) \\ &= (5 + 3)x^2y^2 + (-2 + 5)xy^2 \\ &= 8x^2y^2 + 3xy^2 \end{aligned}$$

Note After sufficient practice, we should be able to carry out the addition and subtraction by grouping mentally.

Like terms

7.
$$\begin{aligned} a^2b + 5ab^2 - 4a^2b + 3ab^2 &= (1 - 4)a^2b + (5 + 3)ab^2 \\ &= -3a^2b + 8ab^2 \end{aligned}$$

Like terms

▶ ***Quick check*** Perform the indicated addition and subtraction:
$3a - 2b + a + 5b$; $\quad 3a^2 + 5a - 2a^2 - a$ ■

Grouping symbols

In chapter 1, we learned that any quantity enclosed within grouping symbols is treated as a single number. We are now going to use the distributive property to remove grouping symbols such as ( ), [ ], and { }. Consider the following examples:

1. The quantity $(2a + 3b)$ can be written as $1 \cdot (2a + 3b)$. Applying the distributive property, we have

$$1(2a + 3b) = 1 \cdot 2a + 1 \cdot 3b = 2a + 3b$$

2. The quantity $+(2a + 3b)$ can be written as $(+1) \cdot (2a + 3b)$ giving

$$(+1)(2a + 3b) = (+1) \cdot 2a + (+1) \cdot 3b = 2a + 3b$$

3. The quantity $-(2a + 3b)$ can be written as $(-1) \cdot (2a + 3b)$ giving

$$(-1)(2a + 3b) = (-1) \cdot 2a + (-1) \cdot 3b = -2a - 3b$$

Removing grouping symbols

1. If an expression inside a grouping symbol is preceded by no symbol or by a "+" sign, the grouping symbol can be dropped and the enclosed terms remain unchanged.
2. If an expression inside a grouping symbol is preceded by a "−" sign, when the grouping symbol is dropped, we change the sign of each enclosed term.

Example 2–3 D

Remove all grouping symbols and perform the indicated addition or subtraction.

1. $(3x^2 + 2x + 5) + (4x^2 + 3x + 6)$ — Remove grouping symbols

$= 3x^2 + 2x + 5 + 4x^2 + 3x + 6$ — Enclosed terms remain unchanged

$= (3x^2 + 4x^2) + (2x + 3x) + (5 + 6)$ — Associative and commutative properties

$= (3 + 4)x^2 + (2 + 3)x + 11$ — Distributive property

$= 7x^2 + 5x + 11$ — Combine numerical coefficients

2. $(3x^2 - x + 4) - (2x^2 - 5x - 7)$ — Remove grouping symbols

$= 3x^2 - x + 4 - 2x^2 + 5x + 7$ — Change the sign of each term contained in the second set of parentheses

$= (3x^2 - 2x^2) + (-x + 5x) + (4 + 7)$ — Associative and commutative properties

$= (3 - 2)x^2 + (-1 + 5)x + 11$ — Distributive property

$= 1x^2 + 4x + 11$ — Combine numerical coefficients

$= x^2 + 4x + 11$ — x^2 is the same as $1x^2$

Note In the following examples, we will mentally add or subtract the like terms.

3. $(a^2 + 2ab + b^2) - (3a^2 - 4ab + b^2)$	Remove grouping symbols
$= a^2 + 2ab + b^2 - 3a^2 + 4ab - b^2$	Change the sign of each term in the second parentheses
$= (a^2 - 3a^2) + (2ab + 4ab) + (b^2 - b^2)$	Associative and commutative properties
$= -2a^2 + 6ab + 0$	Combine like terms
$= -2a^2 + 6ab$	No b^2 is left
4. $(8R^2 - 2R + 3) - (6R^2 + 6R - 1)$	Remove grouping symbols
Like terms	
$= 8R^2 - 2R + 3 - 6R^2 - 6R + 1$	Change the sign of each term in the second parentheses
Like terms	
Like terms	
$= 2R^2 - 8R + 4$	Combine like terms

▶ ***Quick check*** Remove all grouping symbols and perform the indicated addition or subtraction: $(5x^2 + 2x - 1) - (3x^2 - 4x + 3)$ ■

There are many situations where there will be grouping symbols within grouping symbols. In these situations, *it is usually easier to remove the innermost grouping symbol first.*

■ *Example 2–3 E*

Remove all grouping symbols and perform the indicated addition or subtraction.

1. $2x - [y + (x - z)]$	
$= 2x - [y + x - z]$	Remove parentheses first
$= 2x - y - x + z$	Next remove brackets
$= x - y + z$	Combine like terms
2. $2a - [3b - (2a - b)]$	
$= 2a - [3b - 2a + b]$	Remove parentheses first
$= 2a - [4b - 2a]$	Combine like terms within the brackets
$= 2a - 4b + 2a$	Remove brackets
$= 4a - 4b$	Combine like terms

Note After removing the parentheses, we added the like terms before removing the brackets. *Simplify inside grouping symbols as much as possible before going on.*

3. $(3R - 2S) - [5R - (R - S)]$	
$= 3R - 2S - [5R - R + S]$	Remove both sets of parentheses
$= 3R - 2S - [4R + S]$	Combine like terms within brackets
$= 3R - 2S - 4R - S$	Remove brackets
$= -R - 3S$	Combine like terms

Note There were two separate sets of grouping symbols here. As long as they are separate, we may remove both sets at the same time.

▶ ***Quick check*** Remove all grouping symbols and perform the indicated addition or subtraction: $5a - \{2a + [5b - 3a]\}$ ■

Mastery points

Can you

- Identify like terms?
- Add and subtract algebraic expressions?
- Remove grouping symbols?

Exercise 2–3

For the groups of terms, write like or unlike. See example 2–3 B.

Example $4a^2b^2$ and $4a^3b^3$

Solution Both contain the same variables but are **unlike** because the exponents of the respective variables are not the same.

1. $3a, -2a$
2. $5x, 7x$
3. $4a^2, a^2$
4. $b^3, -2b^3$
5. $2a^2, 2a^3$
6. $4x, 4x^2$

Perform the indicated addition and subtraction. See examples 2–3 A, B, and C.

Examples $3a - 2b + a + 5b$ | $3a^2 + 5a - 2a^2 - a$

Solutions

$= (3a + a) + (-2b + 5b)$	$= (3a^2 - 2a^2) + (5a - a)$	Commutative and associative properties
$= (3 + 1)a + (-2 + 5)b$	$= (3 - 2)a^2 + (5 - 1)a$	Distributive property
$= 4a + 3b$	$= 1a^2 + 4a$	Combine numerical coefficients
	$= a^2 + 4a$	

7. $2x + x + 6x$
8. $8y - y + 2y$
9. $4a - 2b + 9a + 4b$
10. $a + 4b + 6a - 8b$
11. $3x + 4x + 7x$
12. $2a^2b - 4a^2b + 6a^2b$
13. $4ab + 11ab - 10ab - 8ab$
14. $d^2 + d - 3d^2 + d^4 + 4d^2$
15. $5x + x^2 - x + 6x^2$
16. $5x^2y - 3xy + 5y + 6xy - x^2y$
17. $a^2b - b^3 - ab^2 + 2a^3 - 5ab^2$
18. $x + 2x^2 - 5 + x^3 - 2x - 2x^2$
19. $3a + b + 2a - 5c - b - 2x^2 + 8a$
20. $3a + 8a - 6a + 9a$
21. $3a + 8b - 6a - 17b$
22. $28ab - 73ab + ab + 11ab - 9ab$
23. $4x^2 - y^2 - x^2 + 12y^2$
24. $5a + 4a^2 - 2a - a^2$
25. $x^2 + 5x - 8x + 2x^2$
26. $8ab + 7a^2b + 6a^2b^2 - 4a^2b$
27. $x^2y^2 + 9xy - 2x^2y - 4xy$
28. $a^2b + 8ab + 3a^2b - 4a^2b^2$
29. $x^2 + 5x - 6 + 7x^2 - 3x + 7$
30. $6a^2 - 5a + 3 - 2a^2 - 4a + 8$

Remove all grouping symbols and combine like terms. See examples 2–3 D and E.

Example $(5x^2 + 2x - 1) - (3x^2 - 4x + 3)$

Solution $= 5x^2 + 2x - 1 - 3x^2 + 4x - 3$ — Change the sign of each term in the second parentheses
$= (5x^2 - 3x^2) + (2x + 4x) + (-1 - 3)$ — Commutative and associative properties
$= 2x^2 + 6x - 4$ — Combine like terms

Example $5a - \{2a + [5b - 3a]\}$

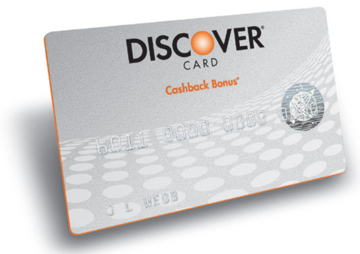
Solution $= 5a - \{2a + 5b - 3a\}$ — Remove brackets
$= 5a - \{-a + 5b\}$ — Combine like terms within braces
$= 5a + a - 5b$ — Remove braces
$= 6a - 5b$ — Combine like terms

31. $(2x + 3y) + (x + 5y)$
32. $(4a - b) + (3a + 2b)$
33. $(5x + y) - (3x - 2y)$
34. $(7x - 3y) - (5x - 6y)$
35. $(3a - b + 4c) - (a - 2b - c)$
36. $(4x - 3y - 2z) - (3x - 4y - z)$
37. $(8x + 3y - 4z) - (6x - y - 4z)$
38. $(7a - b - 3c) - (5a - 4b + 3c)$
39. $(2x^2y - xy^2 + 7xy) + (xy^2 - 5x^2y + 8xy)$
40. $(5x^2 - y^2) - (6x^2 - 3y^2) - (8x^2 + 2y^2)$
41. $(8a^3 - 2a^2b + 4ab^2 - 6b^3) - (4a^3 - 3a^2b - 2ab^2 - b^3)$
42. $(13a - 24bc) + (46bc - 16a - 26d)$
43. $(48a + 3b) - (-22a - 6b)$
44. $(3x^2y - 6xy + 32z) + (7xy - 3x^2y)$
45. $(8xy + 9y^2z) - (13xy - 14yz)$
46. $(18a + 31b) - (23a - 14bc)$
47. $(a - 3b + 2) - (a + 5b - 8)$
48. $(2x + 6z - 10y) - (8y + 3z - 6x + 4)$
49. $(3a - 2b) - (a + 4b) - (-a + 3b)$
50. $(5xy - y) - (3yz + 2xy) + (3y - 4xy)$
51. $(7x^2 - 2y) + (3z - 4y) - (4x^2 - 6y)$
52. $2x - [3x - (5x - 3)]$
53. $x - 1 + [2x - (x - 1)]$
54. $3a + [2a - (a - b)]$
55. $5x - [4a + 3b + (x - 2y)]$
56. $2a - [a - b - (3a + 2b)]$
57. $5a - (a + b) - [2a - b - (3a + 5b)]$
58. $x - [y + (2x - 3y)] + [2x - y]$
59. $-[4a + 7b - (3a + 5b)]$
60. $6x - \{5a + y + (4x - 7y)\}$
61. $2a + [a - (b - c)] - [2a - (b - c)]$
62. $3x - [6x - (4x - 3y)] - [4y - 3x]$

Review exercises

Write an algebraic expression for each of the following. See section 2–1.

1. The product of x and 3
2. 6 times the sum of a and 7
3. y decreased by 2 and that difference divided by 4
4. A number multiplied by 5
5. A number diminished by 12
6. A number divided by 8 and that quotient decreased by 9



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2–4 ■ The addition and subtraction property of equality

Equations

An **equation** is a statement of equality. If two expressions represent the same number, then placing an equality sign, =, between them forms an equation. The following diagram is used to show the parts of an equation.

$$\underbrace{3x - 7}_{\text{Left member of the equation}} \underset{\substack{\uparrow \\ \text{Equality sign}}}{=} \underbrace{2x + 5}_{\text{Right member of the equation}}$$

A **mathematical statement** is a mathematical sentence that can be labeled true or false. $2 + 3 = 5$ is a true statement, and $3 + 4 = 8$ is a false statement.

Other mathematical sentences, such as

$$2 + x = 8 \text{ and } 4 - x = 7,$$

cannot be labeled as true or false. Such sentences are called **open sentences.** The truth or falsity of the sentence is "open" since we do not know the value that the variable represents.

Solution set

A replacement value for the variable that forms a true statement is called a **root,** or a **solution,** of the equation. We say that a root of the given equation *satisfies* that equation. The **solution set** is the set of all values for the variable that cause the equation to be a true statement.

To check the solution of an equation

1. **Substitute:** Replace the variable in the original equation with the solution.
2. **Order of operations:** Perform the indicated operations.
3. **True statement:** If step 2 produces a true statement, the solution is correct.

■ Example 2–4 A

Determine if the given value is a solution of the equation.

1. $2 + x = 8$ when $x = 6$

If we replace x by 6 in the equation and simplify,

$$\begin{aligned} 2 + x &= 8 \\ 2 + (6) &= 8 \\ 8 &= 8 \end{aligned}$$

the equation is true. Then 6 is the root of the equation. The only solution to this equation is 6, and the solution set would then be $\{6\}$.

2. $4 - x = 7$ when $x = -3$

If we replace x by -3 in the equation and simplify,

$$\begin{aligned} 4 - x &= 7 \\ 4 - (-3) &= 7 \\ 7 &= 7 \end{aligned}$$

the equation is true, and -3 is the root of the equation. The solution set is $\{-3\}$.

▶ ***Quick check*** Determine if the equation $3x + 3 = 6$ is true when $x = 1$. Determine if the equation $2x - 1 = 3$ is true when $x = 4$. ■

Types of equations

An equation that is true for some values of the variable and false for other values of the variable is called a **conditional equation.** The equation $2 + x = 8$ is a conditional equation since it is true when $x = 6$ and false otherwise.

If the equation is true for every permissible value of the variable, it is called an **identical equation,** or **identity.** For example,

$$2(x + 3) = 2x + 6$$

is true for any real number replacement for x and is thus an identity. Properties such as

$$a + (b + c) = (a + b) + c \text{ and } a \cdot b = b \cdot a$$

are further examples of identities.

We will study conditional equations in this chapter. These equations will be *first-degree conditional* equations, also called **linear equations.** A linear equation is an equation where the exponent of the unknown is 1 and the solution set will contain at most one root. The equations $2 + x = 8$ and $4 - x = 7$ are linear equations since the variable, x, is to the first power. There is only one replacement value for the variable that will satisfy each equation. That is, for $2 + x = 8$, the solution set is $\{6\}$ since $2 + (6) = 8$, and for $4 - x = 7$, the solution set is $\{-3\}$ since $4 - (-3) = 7$.

Equivalent equations

So far we have looked at such linear equations as $2 + x = 8$ and $4 - x = 7$ for which the solution sets could be determined by inspection. Unfortunately, the majority of equations cannot be solved by inspection. We must develop a procedure for finding the roots.

If we wish to solve a more complicated equation, such as

$$2 + x + 3x - 4 = 2x + 4 + x,$$

the solution is not so obvious. To solve such an equation, we go through a series of steps whereby we form equations that are equivalent to the original equation until we have the equation in the form $x = n$, n being some real number. These equations that we form are called **equivalent equations.** *Equivalent equations are equations whose solution set is the same.*

■ ***Example 2–4 B***

The following are equivalent equations whose solution set is $\{6\}$.

1. $2 + x + 3x - 4 = 2x + 4 + x$ **2.** $4x - 2 = 3x + 4$

3. $x - 2 = 4$ **4.** $x = 6$ ■

Addition and subtraction property of equality

Since an equation is a statement of equality between two expressions, identical quantities added to or subtracted from each expression will produce an equivalent equation. We can state this property as follows:

Addition and subtraction property of equality

For any algebraic expressions a, b, and c,

$$\text{if } a = b, \text{ then } a + c = b + c \text{ and } a - c = b - c$$

Concept

We can add or subtract the same quantity in each member of an equation and the result will be an equivalent equation.

Consider the equation $x - 2 = 4$. To determine the solution by means other than inspection, we want to form an equivalent equation of the form $x = n$. This can be done by applying the addition and subtraction property of equality. We add 2 to both members of the equation and then simplify each member separately.

$$\begin{aligned} x - 2 &= 4 \\ x - 2 + 2 &= 4 + 2 \\ x &= 6 \end{aligned}$$

The root is 6, and the solution set is represented as $\{6\}$.

Check: $(6) - 2 = 4$ Substitute
$4 = 4$ True

■ **Example 2–4 C**

Find the solution set and check the answer.

1. $x - 5 = 7$

$x - 5 + 5 = 7 + 5$ Add 5 to both members

$x + 0 = 12$ Additive inverse

$x = 12$ Solution

Check: $(12) - 5 = 7$ Substitute
$7 = 7$ True

The solution set is $\{12\}$.

2. $x + 4 = 12$

$x + 4 - 4 = 12 - 4$ Subtract 4 from both members

$x = 8$ Solution

Check: $(8) + 4 = 12$ Substitute
$12 = 12$ True

The solution set is $\{8\}$.

Note Subtracting 4 is the same as adding -4. Either method may be used, but we must remember to perform the operation to *both* members of the equation.

3. $x - 8 = -11$

$x - 8 + 8 = -11 + 8$ Add 8 to both members

$x = -3$ Solution

Check: $(-3) - 8 = -11$ Substitute
$-11 = -11$ True

The solution set is $\{-3\}$.

4. $2 = x + 7$

$2 - 7 = x + 7 - 7$ Subtract 7 from both members

$-5 = x$ Solution

Check:

$2 = (-5) + 7$ Substitute

$2 = 2$ True

The solution set is $\{-5\}$.

▶ ***Quick check*** Find the solution set for $x - 7 = 12$ and check the answer.

Symmetric property of equality

The **symmetric property of equality** is also useful in finding the solution set of equations.

Symmetric property of equality

If $a = b$, then $b = a$

Concept

This property allows us to interchange the right and left members of the equation.

In example 2–4 C–4, instead of leaving the equation as $-5 = x$, we could use the symmetric property and write the equation as $x = -5$.

We can see that our goal in solving an equation is to isolate the unknown in one member of the equation and to place everything else in the other member. This forms an equation of the type $x = n$. When the unknown appears in both members of the equation, we use the addition and subtraction property of equality to form an equivalent equation where the unknown appears only in one member of the equation.

Example 2–4 D

Find the solution set and check the answer.

1. $5x - 4 = 4x + 3$

$5x - 4 - 4 = 4x - 4x + 3$ Subtract $4x$ from both members

$x - 4 = 3$

$x - 4 + 4 = 3 + 4$ Add 4 to both members

$x = 7$

Check: $5(7) - 4 = 4(7) + 3$ Substitute

$35 - 4 = 28 + 3$ Order of operations

$31 = 31$ (True) Solution checks

The solution set is $\{7\}$.

2. $6x - 4 = 7x + 2$

$6x - 6x - 4 = 7x - 6x + 2$ Subtract $6x$ from both members

$-4 = x + 2$

$-4 - 2 = x + 2 - 2$ Subtract 2 from both members

$-6 = x$

$x = -6$ Symmetric property

Check: $6(-6) - 4 = 7(-6) + 2$ Substitute

$-36 - 4 = -42 + 2$ Order of operations

$-40 = -40$ (True) Checks

The solution set is $\{-6\}$.

3.

$$\begin{aligned} -2x - 5 &= -3x + 4 \\ -2x + 3x - 5 &= -3x + 3x + 4 && \text{Add } 3x \text{ to both members} \\ x - 5 &= 4 \\ x - 5 + 5 &= 4 + 5 && \text{Add 5 to both members} \\ x &= 9 \end{aligned}$$

Note A good habit for us to develop is to form equivalent equations in which the unknown appears only in the member of the equation that has the greater coefficient of the unknown. This will ensure a positive coefficient for the unknown.

Check:

$$\begin{aligned} -2(9) - 5 &= -3(9) + 4 && \text{Substitute} \\ -18 - 5 &= -27 + 4 && \text{Order of operations} \\ -23 &= -23 \quad \text{(True)} && \text{Solution checks} \end{aligned}$$

The solution set is $\{9\}$.

▶ ***Quick check*** Find the solution set for $4x - 2 = 3x + 5$ and check the solution. ■

Sometimes it is necessary to use the associative, commutative, and distributive properties to perform indicated operations in one or both members of an equation. This will *simplify* the equation before the addition and subtraction property of equality is used. Consider the examples that follow.

■ Example 2–4 E

Find the solution set.

1.

$$\begin{aligned} 5x - 4 + 2x &= 6x - 6 + 11 \\ 7x - 4 &= 6x + 5 && \text{Simplify} \\ 7x - 6x - 4 &= 6x - 6x + 5 && \text{Subtract } 6x \text{ from both members} \\ x - 4 &= 5 \\ x - 4 + 4 &= 5 + 4 && \text{Add 4 to both members} \\ x &= 9 \end{aligned}$$

The solution set is $\{9\}$.

2.

$$\begin{aligned} 3x &= 2(2x - 4) \\ 3x &= 4x - 8 && \text{Simplify} \\ 3x - 3x &= 4x - 3x - 8 && \text{Subtract } 3x \text{ from both members} \\ 0 &= x - 8 \\ 0 + 8 &= x - 8 + 8 && \text{Add 8 to both members} \\ 8 &= x \\ x &= 8 && \text{Symmetric property} \end{aligned}$$

The solution set is $\{8\}$.

3.

$$\begin{aligned} 3(3x - 1) + 4 &= 2(4x + 3) \\ 9x - 3 + 4 &= 8x + 6 && \text{Simplify} \\ 9x + 1 &= 8x + 6 && \text{Simplify} \\ 9x - 8x + 1 &= 8x - 8x + 6 && \text{Subtract } 8x \text{ from both members} \\ x + 1 &= 6 \\ x + 1 - 1 &= 6 - 1 && \text{Subtract 1 from both members} \\ x &= 5 \end{aligned}$$

The solution set is $\{5\}$.

▶ ***Quick check*** Find the solution set for $5x + 2x - 4 = 6x + 7$ and for $3(2x + 1) = x + 4x - 2$ ■

Problem solving

We are now ready to combine our ability to write an expression and our ability to solve an equation and apply them to solve a word problem. While there is no standard procedure for solving a word problem, the following guidelines should be useful.

Solving word problems

1. Read the problem carefully. Determine useful prior knowledge and note what information is given and what information we are asked to find.
2. Choose a variable to represent one of the unknowns and then express other unknowns in terms of it.
3. Use the given conditions in the problem and the unknowns from step 2 to write an algebraic equation.
4. Solve the equation for the unknown. Relate this answer to any other unknowns in the problem.
5. Check your results in the original statement of the problem.

Example 2–4 F

Solve the following word problems by setting up an equation and solving it.

1. A number increased by 16 gives 24. Find the number.

Let n = the number we are looking for. The key words to use are "increased by," which means *add,* and "gives," which means *equals.* The equation is then

a number	increased by	16	gives	24
n	$+$	16	$=$	24

$n + 16 = 24$

$n = 8$ Subtract 16 from each member

The number is 8.

2. Joan earned \$15 less than Mary did last week. If Joan earned \$342, how much did Mary earn?

Let d = the amount that Mary earned last week. The key words are "less than." Since Joan earned \$15 less than Mary, the equation is given by

Mary's salary	less \$15	is	Joan's salary
d	-15	$=$	342

$d - 15 = 342$

$d = 357$ Add 15 to each member

Mary earned \$357 last week. ■

Mastery points

Can you

- Determine if a given number is a root of an equation?
- Use the addition and subtraction property of equality?
- Simplify equations?
- Solve for an unknown?
- Check your answer?

Exercise 2–4

Determine if the given value is a solution of the equation. See example 2–4 A.

Examples $3x + 3 = 6; \{1\}$

Solutions

$$\begin{aligned} 3(1) + 3 &= 6 && \text{Substitute} \\ 3 + 3 &= 6 && \text{Order of operations} \\ 6 &= 6 && \text{Checks} \end{aligned}$$

$2x - 1 = 3; \{4\}$

$$\begin{aligned} 2(4) - 1 &= 3 && \text{Substitute} \\ 8 - 1 &= 3 && \text{Order of operations} \\ 7 &= 3 && \text{Does not check} \end{aligned}$$

1. $4 + x = 8; \{4\}$
2. $3 - x = 4; \{-1\}$
3. $x + 7 = 10; \{3\}$
4. $3x - 2 = 4; \{2\}$
5. $8x + 6 = 2x - 6; \{-2\}$
6. $\frac{4}{5}x + 2 = 10; \{10\}$
7. $7x - 3 = 2x + 2; \{-2\}$
8. $3x + 2 = 5x - 1; \left\{\frac{3}{2}\right\}$
9. $2(x - 1) = 4x + 5; \left\{-\frac{7}{2}\right\}$
10. $5x - 1 = 11x - 1; \{0\}$
11. $\frac{x}{5} - 2 = 3x + 1; \{1\}$
12. $\frac{2x}{3} - 1 = \frac{x}{4} + 3; \{2\}$

Find the solution set by using the addition and subtraction property of equality. Check each solution. See examples 2–4 C and D.

Examples $x - 7 = 12$

Solutions

$$\begin{aligned} x - 7 + 7 &= 12 + 7 && \text{Add 7} \\ x &= 19 \end{aligned}$$

Check:

$$\begin{aligned} (19) - 7 &= 12 && \text{Substitute} \\ 12 &= 12 \quad \text{(True)} && \text{Checks} \end{aligned}$$

The solution set is $\{19\}$.

$4x - 2 = 3x + 5$

$$\begin{aligned} 4x - 3x - 2 &= 3x - 3x + 5 && \text{Subtract } 3x \\ x - 2 &= 5 \\ x - 2 + 2 &= 5 + 2 && \text{Add 2} \\ x &= 7 \end{aligned}$$

Check:

$$\begin{aligned} 4(7) - 2 &= 3(7) + 5 && \text{Substitute} \\ 28 - 2 &= 21 + 5 && \text{Order of operations} \\ 26 &= 26 \quad \text{(True)} && \text{Checks} \end{aligned}$$

The solution set is $\{7\}$.

13. $x - 4 = 12$
14. $y - 7 = 11$
15. $a + 5 = 2$
16. $b + 5 = 7$
17. $y - 6 = -8$
18. $5 = x + 7$
19. $9 = x + 14$
20. $a - 5 = -2$
21. $-10 = x - 4$
22. $a - 18 = -14$
23. $b + 7 = 0$
24. $y - 14 = 0$
25. $3x - 4 = 2x + 10$
26. $6x - 5 = 5x + 11$
27. $b + 4 = 2b + 5$
28. $-y - 6 = -2y + 1$
29. $-z - 8 = -2z - 4$
30. $5 - 3x = 7 - 4x$
31. $9 - 7a = 14 - 6a$
32. $3a - 5 = 2a - 2$

Find the solution set. See example 2–4 E.

Examples $5x + 2x - 4 = 6x + 7$

Solutions

$$\begin{aligned} 7x - 4 &= 6x + 7 && \text{Combine like terms} \\ 7x - 6x - 4 &= 6x - 6x + 7 && \text{Subtract } 6x \\ x - 4 &= 7 \\ x - 4 + 4 &= 7 + 4 && \text{Add 4} \\ x &= 11 \end{aligned}$$

The solution set is $\{11\}$.

$3(2x + 1) = x + 4x - 2$

$$\begin{aligned} 6x + 3 &= 5x - 2 && \text{Simplify} \\ 6x - 5x + 3 &= 5x - 5x - 2 && \text{Subtract } 5x \\ x + 3 &= -2 \\ x + 3 - 3 &= -2 - 3 && \text{Subtract 3} \\ x &= -5 \end{aligned}$$

The solution set is $\{-5\}$.

33. $6a - 3a + 7 = 9a - 5a + 2$
34. $-4x - 2x + 1 = -5x + 7$
35. $7b - 2b + 5 - 4b = 11$
36. $12 = 6x + 3 - 4x - x$
37. $-4 - x = 4x + 2 - 6x$
38. $5(x + 2) = 4(x - 1)$
39. $2(2y - 1) = 3(y + 2)$
40. $5(3x + 2) = 7(2x + 3)$
41. $5x - 4 + x = 5(x - 2)$
42. $3(2x + 1) - 7 = 5x - 4$
43. $(4a + 5) - (2 + 3a) = 8$
44. $(9b + 7) - (8b + 2) = -4$
45. $3(z + 7) - (8 + 2z) = 6$
46. $4(x - 5) - (3x + 4) = -2$
47. $2(a - 3) - (a - 2) = 8$
48. $5(a + 1) - (4a + 3) = 14$
49. $2(3x - 1) + 3(x + 2) = 4(2x + 5)$
50. $3(4x - 5) + 2(x - 4) = 3(5x + 2)$
51. $-2(b + 1) + 3(b - 4) = -5$
52. $-3(x - 2) + 4(x - 5) = -7$

Solve the following problems by setting up an equation and solving for the unknown. See example 2–4 F.

53. A number increased by 11 yields 37. Find the number.

54. If a number is decreased by 16, the result is 52. Find the number.

55. If Gary's age is increased by 4 years, he is 37 years old. How old is Gary now?

56. Harry is 6 years older than Dene. If Dene is 54 years old, how old is Harry?

57. If Jake withdraws \$340 from his savings account, his balance will be \$395. How much does Jake have in his savings account now?

58. Pam deposits \$42.50 in her checking account. If her new balance is \$125.30, how much did she have in her account originally?

59. Mr. Johnson took in \$560 on a given day in his grocery store. If he paid out \$195 to his employees in wages, how much profit did he realize?

60. Marsha can groom 11 more dogs per day than Margaret can. If Marsha can groom 24 dogs per day, how many dogs per day can Margaret groom?

Review exercises

Perform the indicated operations. See sections 1–6 and 1–7.

1. $(-2)(-8)$ **2.** $(-4)(3)$ **3.** $\dfrac{-8}{-8}$ **4.** $\dfrac{6}{6}$ **5.** $\left(\dfrac{1}{3}\right)(3)$ **6.** $\left(-\dfrac{1}{4}\right)(-4)$

2–5 ■ The multiplication and division property of equality

Multiplication and division property of equality

In section 2–4, we used the associative, commutative, and distributive properties to simplify equations. We then used the addition and subtraction property of equality to solve for the unknown. These properties are sufficient to solve many of the equations that we encounter. However we cannot use them to solve such equations as

$$3x = 21 \quad \text{or} \quad \frac{2}{3}x = 12$$

Recall that we want our equation to be of the form $x = n$. This means that the coefficient of x must be 1. To achieve this, we make use of the multiplication and division property of equality.

The multiplication and division property of equality

For any algebraic expressions a, b, and c ($c \neq 0$)

$$\text{if } a = b, \text{ then } a \cdot c = b \cdot c \text{ and } a \div c = b \div c$$

Concept

An equivalent equation is obtained when we multiply or divide both members of an equation by the same nonzero quantity.

The multiplication and division property of equality enables us to multiply or divide both members of an equation by the same nonzero quantity. In the equation $3x = 21$, we use the multiplication and division property of equality to divide both members of the equation by 3. This forms an equivalent equation where x has a coefficient of 1, that is, $x = n$.

$$3x = 21$$

$$\frac{3x}{3} = \frac{21}{3} \quad \text{Divide both members by 3}$$

$$x = 7$$

The solution set is $\{7\}$.

For the equation $\frac{2}{3}x = 12$, recall that when we divide by a fraction, we invert and multiply. Therefore, if the coefficient is a fraction, we will multiply both members of the equation by the **reciprocal,** or the **multiplicative inverse,** of the coefficient.

Multiplicative inverse

The **multiplicative inverse** of a number, also called the **reciprocal** of the number, is such that when we multiply a number times its reciprocal, the answer will be 1.

Multiplicative inverse property

For every real number a, $a \neq 0$,

$$a \cdot \frac{1}{a} = 1$$

Concept

Every real number except zero has a multiplicative inverse, and the product of a number and its multiplicative inverse is always 1.

Note Zero is the only number that does not have a reciprocal. From the zero factor property, we know that zero times any number is zero. Therefore there can be no number such that zero times that number gives 1 as an answer.

$$a \cdot 0 = 0$$

Example 2–5 A

The following examples are illustrations of the multiplicative inverse property, where the second number can be considered the reciprocal of the first, and the first can be considered the reciprocal of the second.

1. $5 \cdot \frac{1}{5} = 1$ **2.** $\frac{1}{2} \cdot 2 = 1$ **3.** $b \cdot \frac{1}{b} = 1, b \neq 0$

4. $\frac{3}{4} \cdot \frac{4}{3} = 1$ 5. $\left(-\frac{5}{7}\right)\left(-\frac{7}{5}\right) = 1$

We will now use the multiplicative inverse property to solve the equation $\frac{2}{3}x = 12$.

$$\frac{2}{3}x = 12$$
$$\frac{3}{2} \cdot \frac{2}{3}x = \frac{3}{2} \cdot 12 \quad \text{Multiply both members by the reciprocal of the coefficient}$$
$$x = 18$$

The solution set is $\{18\}$.

Note In the earlier example $3x = 21$, we could have multiplied by the reciprocal of 3 to solve the equation. That is,

$$3x = 21$$
$$\frac{1}{3} \cdot 3x = \frac{1}{3} \cdot 21 \quad \text{Multiply both members by the reciprocal } \frac{1}{3}$$
$$x = 7$$

Remember that to divide by a number is the same operation as to multiply by the reciprocal of that number.

Example 2–5 B

Find the solution set.

1. $5x = 30$
$$\frac{5x}{5} = \frac{30}{5} \quad \text{Divide both members by the coefficient 5}$$
$$x = 6$$

The solution set is $\{6\}$.

2. $\frac{3}{4}x = 9$
$$\frac{4}{3} \cdot \frac{3}{4}x = \frac{4}{3} \cdot 9 \quad \text{Multiply both members by the reciprocal } \frac{4}{3}$$
$$x = 12$$

The solution set is $\{12\}$.

3. $-x = -10$
$$-1 \cdot x = -10 \quad -1 \text{ is the coefficient}$$
$$\frac{-1 \cdot x}{-1} = \frac{-10}{-1} \quad \text{Divide both members by the coefficient } -1$$
$$x = 10$$

The solution set is $\{10\}$.

4. $6x = 10$
$$\frac{6x}{6} = \frac{10}{6} \quad \text{Divide both members by the coefficient 6}$$
$$x = \frac{5}{3} \quad \text{Reduce the fraction}$$

The solution set is $\left\{\frac{5}{3}\right\}$.

5. $\dfrac{x}{4} = 6$ — We can rewrite the left member to show that the coefficient is $\frac{1}{4}$

$\dfrac{1}{4}x = 6$

$4 \cdot \dfrac{1}{4}x = 4 \cdot 6$ — Multiply both members by the reciprocal 4

$x = 24$

The solution set is $\{24\}$.

6. $1.2x = 4.8$

$\dfrac{1.2x}{1.2} = \dfrac{4.8}{1.2}$ — Divide both members by 1.2

$x = 4$

The solution set is $\{4\}$.

▶ ***Quick check*** Find the solution set of the equations $5x = 35$, $\frac{2}{3}x = 8$, and $1.7x = 10.2$ ■

Problem solving

Now we will translate some word statements into equations and solve the resulting equations.

■ Example 2–5 C

Write an equation for each problem and then solve the equation.

1. When a number is multiplied by -6, the result is 48. Find the number.

Let n = the number for which we are looking. The formation of the equation would be as follows:

a number	multiplied by (-6)	result is	48
n	$\cdot(-6)$	$=$	48

$n \cdot (-6) = 48$

$\dfrac{n(-6)}{-6} = \dfrac{48}{-6}$ — Divide both members by -6

$n = -8$

The number is -8.

2. Alice makes \$4.50 per hour. If her pay was \$108, how many hours did she work?

Let n = the number of hours that she worked. The formation of the equation would be as follows:

hourly rate	times	number of hours worked	gives	total pay
(4.50)		n	$=$	108

$(4.50) \cdot n = 108$

$\dfrac{(4.50)n}{4.50} = \dfrac{108}{4.50}$ — Divide both members by 4.50

$n = 24$

Alice worked 24 hours. ■

Mastery points

Can you

- Use the multiplication and division property of equality to form equivalent equations where the coefficient of the unknown is 1?
- Check your answer?

Exercise 2–5

Find the solution set by using the multiplication and division property of equality. Check each solution. See example 2–5 B.

Examples $5x = 35$

Solutions $\frac{5x}{5} = \frac{35}{5}$ Divide by 5

$x = 7$

The solution set is $\{7\}$.

Check:

$5(7) = 35$ Substitute

$35 = 35$ (True) Checks

Examples $\frac{2}{3}x = 8$

Solutions $\frac{3}{2} \cdot \frac{2}{3}x = \frac{3}{2} \cdot 8$ Multiply by $\frac{3}{2}$

$x = 12$

The solution set is $\{12\}$

Check:

$\frac{2}{3}(12) = 8$ Substitute

$8 = 8$ (True) Checks

Examples $1.7x = 10.2$

Solutions $\frac{1.7x}{1.7} = \frac{10.2}{1.7}$ Divide by 1.7

$x = 6$

The solution set is $\{6\}$.

Check:

$1.7(6) = 10.2$ Substitute

$10.2 = 10.2$ (True) Checks

1. $2x = 8$
2. $3x = 18$
3. $6x = 36$
4. $9x = 45$
5. $\frac{3}{4}x = 12$
6. $\frac{2}{5}x = 10$
7. $\frac{1}{7}x = 5$
8. $\frac{1}{5}x = 9$
9. $\frac{3}{2}x = 18$
10. $14 = \frac{7}{3}x$
11. $5x = -15$
12. $-8 = 2x$
13. $-24 = 6x$
14. $-5x = 30$
15. $-4x = -28$
16. $-30 = -6x$
17. $-x = 4$
18. $-x = -11$
19. $6x = 14$
20. $5x = 9$
21. $4x = 6$
22. $3x = -8$
23. $5x = 0$
24. $0 = 7x$
25. $-3x = 0$
26. $-2x = 0$
27. $\frac{x}{3} = 5$
28. $\frac{x}{4} = 8$
29. $\frac{x}{-2} = 7$
30. $-2 = \frac{x}{-3}$
31. $2.6x = 10.4$
32. $3.1x = 21.7$
33. $-4.8x = 33.6$
34. $-7.1x = 35.5$
35. $-42.9 = -3.9x$
36. $(0.4)x = 7.2$
37. $(0.3)x = -7.8$
38. $\frac{5}{7}x = 8$
39. $\frac{3}{8}x = 14$
40. $\frac{2}{9}x = 11$

Write an equation for each exercise and then solve the equation. See example 2–5 C.

41. When a number is multiplied by 6, the result is 54. Find the number.
42. When a number is multiplied by -4, the result is 36. Find the number.
43. When a number is divided by 9, the result is -7. Find the number.
44. When a number is divided by -8, the result is -8. Find the number.

$\boxed{45.}$ Nancy worked for 30 hours and received \$135. Find her hourly wage.

46. Adam worked for 14 hours and received \$52.50. Find his hourly wage.

47. Four friends shared equally in the expenses for a party. If each person's share was \$32.50, what was the total cost of the party?

48. Six friends shared equally in the cost of dinner. If the cost of the dinner was \$51, what was each person's share?

49. If $\frac{3}{4}$ of a number is 48, find the number.

50. If $\frac{2}{3}$ of a number is 26, find the number.

Review exercises

Perform all indicated operations. See section 2–3.

1. $3x + 2x + 1 - 3$ **2.** $7x - 5x - 3 + 4$ **3.** $8x - 5 + 4x + 7$

4. $6x + 3 - 3x - 8$ **5.** $2(3x + 1) + 4x - 3$ **6.** $3(x - 1) + 2(x + 2)$

2–6 ■ Solving linear equations

Review of properties

We now are ready to combine the properties from the previous sections to help us solve more involved equations. The process consists of forming equivalent equations until we have our equation in the form of $x = n$. The properties that we will use are the following:

1. We can add or subtract the same number in both members of the equation.
2. We can multiply or divide both members of the equation by the same nonzero number.

If we use these two properties, making sure that both members of the equation are treated in exactly the same manner as we apply each of the properties, we will be forming equivalent equations.

Procedure for solving a linear equation

Using these properties, there are four basic steps to solve a linear equation. We shall now apply the properties to the equation $6(x + 1) = 4x + 10$.

Solving a linear equation

$$6(x + 1) = 4x + 10$$

Step 1 *Simplify each member of the equation.* Perform all indicated addition, subtraction, multiplication, and division. Remove all grouping symbols. In our example, step 1 would be to carry out the indicated multiplication in the left member as follows:

$$\begin{aligned} 6(x + 1) &= 4x + 10 \\ 6x + 6 &= 4x + 10 \end{aligned}$$

Step 2 *Use the addition and subtraction property of equality to form an equivalent equation where all the terms involving the unknown are in one member of the equation.* By subtracting $4x$ from *both* members of the equation, we have

$$\begin{aligned} 6x + 6 &= 4x + 10 \\ 6x - 4x + 6 &= 4x - 4x + 10 \\ 2x + 6 &= 10 \end{aligned}$$

Step 3 *Use the addition and subtraction property of equality to form an equivalent equation where all the terms not involving the unknown are in the other member of the equation.* Subtracting 6 from *both* members of the equation, we have

$$\begin{aligned} 2x + 6 &= 10 \\ 2x + 6 - 6 &= 10 - 6 \\ 2x &= 4 \end{aligned}$$

Step 4 *Use the multiplication and division property of equality to form an equivalent equation where the coefficient of the unknown is 1.* That is, $x = n$. By dividing *both* members of the equation by 2, we have

$$\begin{aligned} 2x &= 4 \\ \frac{2x}{2} &= \frac{4}{2} \\ x &= 2 \end{aligned}$$

The solution set is denoted by $\{2\}$.

Step 5 To check the solution, we substitute the solution in place of the unknown in the original equation. If we get a true statement, we say that the solution "satisfies" the equation.

In the equation $6(x + 1) = 4x + 10$, we found that $x = 2$. We can check the solution by substituting 2 in place of x in the original equation.

$$\begin{aligned} 6[(2) + 1] &= 4(2) + 10 && \text{Substitute} \\ 6[3] &= 8 + 10 && \text{Order of operations} \\ 18 &= 18 \quad \text{(True)} && \text{Solution checks} \end{aligned}$$

We see that $x = 2$ satisfies the equation.

■ Example 2–6 A

Find the solution set and check.

1. $6y + 5 - 7y = 10 - 2y + 3$

$$\begin{aligned} 5 - y &= 13 - 2y && \text{Simplify each member by combining like terms} \\ 5 - y + 2y &= 13 - 2y + 2y && \text{Add } 2y \text{ to both members} \\ 5 + y &= 13 \\ 5 + y - 5 &= 13 - 5 && \text{Subtract 5 from both members} \\ y &= 8 \end{aligned}$$

Check:

$$\begin{aligned} 6(8) + 5 - 7(8) &= 10 - 2(8) + 3 && \text{Substitute} \\ 48 + 5 - 56 &= 10 - 16 + 3 && \text{Order of operations} \\ 53 - 56 &= -6 + 3 \\ -3 &= -3 \quad \text{(True)} && \text{Solution checks} \end{aligned}$$

The solution set is $\{8\}$.

2. $8y + 5 - 7y = 10 - 2y + 3$

$$\begin{aligned} 5 + y &= 13 - 2y && \text{Combine like terms} \\ 5 + y + 2y &= 13 - 2y + 2y && \text{Add } 2y \\ 5 + 3y &= 13 && \text{Combine like terms} \\ 5 + 3y - 5 &= 13 - 5 && \text{Subtract 5} \\ 3y &= 8 && \text{Combine like terms} \\ \frac{3y}{3} &= \frac{8}{3} && \text{Divide by 3} \\ y &= \frac{8}{3} \end{aligned}$$

Check:

$$\begin{aligned} 8\left(\frac{8}{3}\right) + 5 - 7\left(\frac{8}{3}\right) &= 10 - 2\left(\frac{8}{3}\right) + 3 && \text{Substitute } \frac{8}{3} \text{ for } y \\ \frac{64}{3} + \frac{15}{3} - \frac{56}{3} &= \frac{30}{3} - \frac{16}{3} + \frac{9}{3} && \text{Multiply, change to common denominator} \\ \frac{64 + 15 - 56}{3} &= \frac{30 - 16 + 9}{3} && \text{Add and subtract in numerators} \\ \frac{23}{3} &= \frac{23}{3} \quad \text{(True)} && \text{Solution checks} \end{aligned}$$

The solution set is $\left\{\frac{8}{3}\right\}$.

3. $4(5x - 2) + 7 = 5(3x + 1)$

$$\begin{aligned} 20x - 8 + 7 &= 15x + 5 && \text{Distributive property} \\ 20x - 1 &= 15x + 5 && \text{Combine like terms} \\ 20x - 15x - 1 &= 15x - 15x + 5 && \text{Subtract } 15x \\ 5x - 1 &= 5 \\ 5x - 1 + 1 &= 5 + 1 && \text{Add 1} \\ 5x &= 6 \\ \frac{5x}{5} &= \frac{6}{5} && \text{Divide by 5} \\ x &= \frac{6}{5} \end{aligned}$$

Check:

$$4\left[5\left(\frac{6}{5}\right) - 2\right] + 7 = 5\left[3\left(\frac{6}{5}\right) + 1\right] \quad \text{Substitute } \frac{6}{5} \text{ for } x$$
$$4[6 - 2] + 7 = 5\left[\frac{18}{5} + 1\right] \quad \text{Order of operations}$$
$$4[4] + 7 = 5\left[\frac{18}{5} + \frac{5}{5}\right]$$
$$16 + 7 = 5\left[\frac{23}{5}\right]$$
$$23 = 23 \text{ (True)} \quad \text{Solution checks}$$

The solution set is $\left\{\frac{6}{5}\right\}$.

▶ ***Quick check*** Find the solution set for $5x + 2(x - 1) = 4 - 3x$ and check. ■

At this point, we will no longer show the check of our solution, but you should realize that a check of your solution is an important final step.

The following equations contain several fractions. When this occurs, it is usually easier to *clear the equation of all fractions.* We do this by multiplying both members of the equation by the least common denominator of all the fractions. Clearing all fractions is considered a means of simplifying the equation and will be done as a first step when necessary. Equations containing fractions will be studied more completely in chapter 6.

■ Example 2–6 B

Find the solution set.

1.

$$\frac{1}{4}x + 2 = \frac{1}{2}$$
$$4\left(\frac{1}{4}x + 2\right) = 4\left(\frac{1}{2}\right) \quad \text{The least common denominator of the fractions is 4, multiply both members by 4}$$
$$4\left(\frac{1}{4}x\right) + 4(2) = 4\left(\frac{1}{2}\right) \quad \text{Simplify (distributive property)}$$
$$x + 8 = 2 \quad \text{All fractions have been cleared}$$
$$x + 8 - 8 = 2 - 8 \quad \text{Subtract 8}$$
$$x = -6$$

The solution set is $\{-6\}$.

2.

$$\frac{5}{6}x - \frac{2}{3} = \frac{3}{4}x + 2$$
$$12\left(\frac{5}{6}x - \frac{2}{3}\right) = 12\left(\frac{3}{4}x + 2\right) \quad \text{The least common denominator of the fractions is 12; multiply by 12}$$
$$12\left(\frac{5}{6}x\right) - 12\left(\frac{2}{3}\right) = 12\left(\frac{3}{4}x\right) + 12(2) \quad \text{Simplify, distributive property}$$
$$10x - 8 = 9x + 24$$
$$10x - 9x - 8 = 9x - 9x + 24 \quad \text{Subtract } 9x$$
$$x - 8 = 24$$
$$x - 8 + 8 = 24 + 8 \quad \text{Add 8}$$
$$x = 32$$

The solution set is $\{32\}$. ■

Mastery points

Can you

- Solve linear equations?
- Check your answers?

Exercise 2–6

Find the solution set of the following equations, and check the solution. See examples 2–6 A and B.

Example $5x + 2(x - 1) = 4 - 3x$

Solution

$$
\begin{aligned}
5x + 2x - 2 &= 4 - 3x && \text{Simplify (distributive property)}\\
7x - 2 &= 4 - 3x && \text{Combine like terms}\\
7x + 3x - 2 &= 4 - 3x + 3x && \text{Add } 3x\\
10x - 2 &= 4 && \text{Combine like terms}\\
10x - 2 + 2 &= 4 + 2 && \text{Add 2}\\
10x &= 6 && \text{Combine like terms}\\
\frac{10x}{10} &= \frac{6}{10} && \text{Divide by 10 and reduce}\\
x &= \frac{3}{5}
\end{aligned}
$$

Check:

$$
\begin{aligned}
5\left(\frac{3}{5}\right) + 2\left[\left(\frac{3}{5}\right) - 1\right] &= 4 - 3\left(\frac{3}{5}\right) && \text{Substitute } \frac{3}{5} \text{ for } x\\
5\left(\frac{3}{5}\right) + 2\left[\frac{3}{5} - \frac{5}{5}\right] &= 4 - \frac{9}{5} && \text{Order of operations}\\
5\left(\frac{3}{5}\right) + 2\left[\frac{-2}{5}\right] &= \frac{20}{5} - \frac{9}{5}\\
\frac{15}{5} + \frac{-4}{5} &= \frac{11}{5}\\
\frac{11}{5} &= \frac{11}{5} \quad \text{(True)} && \text{Solution checks}
\end{aligned}
$$

The solution set is $\left\{\frac{3}{5}\right\}$.

1. $2x = 4$
2. $3x = 11$
3. $5x = -10$
4. $-2x = 8$
5. $\frac{x}{2} = 18$
6. $\frac{x}{4} = 24$
7. $\frac{3x}{2} = 8$
8. $\frac{5x}{3} = 18$
9. $x + 7 = 11$
10. $x - 4 = 9$
11. $x + 5 = 5$
12. $x - 4 = -4$
13. $3x + 1 = 10$
14. $5x - 2 = 13$
15. $4x + 7 = 7$
16. $6x + 2 = 2$
17. $5x + 2x = x + 6$
18. $2x + (3x - 1) = 4 - x$
19. $2x + 3x - 6x = 4x - 8$
20. $\frac{x}{2} + 7 = 14$
21. $5 - \frac{3x}{5} = 11$
22. $\frac{5x + 2x}{6} = 10$
23. $\frac{1}{2}x + 3 = \frac{3}{4}$
24. $\frac{1}{5}x - 1 = \frac{7}{10}$
25. $\frac{1}{3}x + 2 = \frac{1}{2}x - 1$

26. $\frac{1}{4}x - 3 = \frac{1}{8}x + 1$

27. $\frac{2}{3}x + 5 = \frac{3}{4}$

28. $\frac{3}{5}x - 3 = \frac{3}{10}$

29. $\frac{3}{8}x + \frac{1}{2} = \frac{1}{4}x + 2$

30. $\frac{7}{12}x + 1 = \frac{2}{3}x - 1$

$\boxed{\textbf{31.}}$ $3(2x - 1) = 4x + 3$

32. $5(7x - 3) = 30x + 11$

33. $12x - 8 = 5x + 2$

34. $3(2x + 5) = 4(x - 3)$

$\boxed{\textbf{35.}}$ $8 - 2(3x + 4) = 5x - 16$

36. $(3x + 2) - (2x - 5) = 7$

37. $(7x - 6) - (4 - 3x) = 27$

38. $2(x - 4) - 3(5 - 2x) = 16$

39. $3(2x + 3) = 5 - 4(x - 2)$

40. $6(3x - 2) = 7(x - 3) - 2$

41. $2(x + 5) = 16$

42. $6 = 2(2x - 1)$

43. $2x - (3 - x) = 0$

44. $3(7 - 2x) = 30 - 7(x + 1)$

Find the solution set for the following equations, and check the answer. See examples 2–6 A and B.

45. To convert Celsius temperature to Fahrenheit, we use $F = \frac{9}{5}C + 32$. Find C when (a) $F = 18$, (b) $F = -27$, (c) $F = 2$.

$\boxed{\textbf{46.}}$ The Stefan-Boltzmann Law in metallurgy, which is the temperature scale of radiation pyrometers, is given by $W = KT^4$. Find K when (a) $W = 36$, $T = 2$; $\boxed{\text{(b)}}$ $W = 243$, $T = -3$.

47. The total creep of a metal (E_P) at time t is given by $E_P = E_0 + V_0 t$, where E_0 = original creep, t = time, and V_0 = the original volume. Find V_0 when $E_P = 16$, $E_0 = 9$, and $t = 3$.

48. In a gear system, the speed, in number of revolutions, of two gears and the number of teeth in the gears are related by $S_D \cdot T_D = S_d \cdot T_d$, where S_D is the speed of the driver, T_D is the number of teeth in the driver, S_d is the speed of the driven gear, and T_d is the number of teeth in the driven gear. If
(a) $S_D = 240$, $T_D = 40$, and $S_d = 360$, find T_d;
(b) $S_D = 120$, $S_d = 90$, and $T_d = 18$, find T_D.

Review exercises

Evaluate the following formulas. See section 2–2.

1. $W = I^2R$, $I = 6$ and $R = 3$

2. $S = \frac{1}{2}gt^2$, $g = 32$ and $t = 3$

3. $A = \frac{1}{2}h(b + c)$, $h = 8$, $b = 10$, and $c = 12$

4. $I = prt$; $p = 2{,}000$; $r = 0.06$; and $t = 3$

5. $V = \ell wh$, $\ell = 10$, $w = 4$, and $h = 7$

6. $V = k + gt$, $k = 12$, $g = 16$, and $t = 5$

2–7 ■ Solving literal equations and formulas

Literal equations

Equations that contain two or more variables are called **literal equations.** In a literal equation, we generally solve the equation for one variable in terms of the remaining variables and constants. *The procedure for solving a literal equation is the same as the procedure for solving linear equations.*

Formulas

A formula is a mathematical equation that states the relationship between two or more physical conditions. Consider the formula $d = rt$, which expresses the fact that distance (d) is equal to rate (r) multiplied by the time (t). If we knew how far it was between two cities (d) and we wanted to travel this distance in a certain amount of time (t), then the equation could be solved for the necessary rate (r) to achieve this.

$$d = rt \qquad \text{Divide each member by } t$$
$$\frac{d}{t} = r$$

The equation is now solved for r in terms of d and t. If the distance and rate were known, the equation could be solved for time as follows:

$$d = rt \qquad \text{Divide each member by } r$$
$$\frac{d}{r} = t$$

The equation is now solved for t in terms of d and r.

We observe from this example that we may solve a literal equation or formula for a specified variable. The following list is a restatement of the procedure for solving linear equations that we will now apply to literal equations.

Solving a literal equation or a formula

Step 1 Simplify each member of the equation.

Step 2 Collect all the terms with the variable for which we are solving in one member of the equation. (Addition and subtraction property)

Step 3 Remove any term that is being added to or subtracted from the variable for which we are solving. (Addition and subtraction property)

Step 4 Divide each member of the equation by the coefficient of the variable for which we are solving. (Multiplication and division property)

■ ***Example 2–7 A***

Solve for the specified variable.

1. The volume of a rectangular solid is found by multiplying length (ℓ) times width (w) times height (h), $V = \ell wh$. Solve the equation for h.

$$V = \ell wh \quad \text{Original equation}$$
$$V = (\ell w)h \quad \text{Coefficient of } h \text{ is } \ell w$$
$$\frac{V}{\ell w} = \frac{\ell wh}{\ell w} \quad \text{Divide by } \ell w$$
$$\frac{V}{\ell w} = h \quad \text{Equation is solved for } h \text{ in terms of } V, \ell, \text{ and } w$$
$$h = \frac{V}{\ell w} \quad \text{Symmetric property}$$

2. The simple interest (I) earned on the principal (P) over a time period (t) at an interest rate (r) is given by $I = Prt$. Solve for r.

$$I = Prt \quad \text{Original equation}$$
$$I = (Pt)r \quad Pt \text{ is the coefficient of } r$$
$$\frac{I}{Pt} = \frac{Ptr}{Pt} \quad \text{Divide by } Pt$$
$$\frac{I}{Pt} = r \quad \text{Equation is solved for } r \text{ in terms of } I, P, \text{ and } t$$
$$r = \frac{I}{Pt} \quad \text{Symmetric property}$$

3. If we know the temperature in degrees Fahrenheit (F), the temperature in degrees Celsius (C) can be found by the equation $C = \frac{5}{9}(F - 32)$. Solve the formula for F.

$$C = \frac{5}{9}(F - 32)$$
$$9C = 9 \cdot \frac{5}{9}(F - 32) \quad \text{Clear the fraction}$$
$$9C = 5(F - 32) \quad \text{Apply the distributive property}$$
$$9C = 5F - 160$$
$$9C + 160 = 5F \quad \text{Add 160}$$
$$\frac{9C + 160}{5} = F \quad \text{Divide by 5}$$
$$F = \frac{9C + 160}{5} \quad \text{Symmetric property}$$

If the temperature is given in degrees Celsius, we use this form of the equation to determine the temperature in degrees Fahrenheit.

Note Although we have not stated any restrictions on the variables, it is understood that the values that the variables can take on must be such that no denominator is ever zero. That is, in example 1, $\ell \neq 0$ and $w \neq 0$; in example 2, $P \neq 0$, $t \neq 0$.

▶ ***Quick check*** Solve $P = 2\ell + 2w$ for ℓ. ■

Whether we are solving for x in a linear equation or a literal equation, the procedure is the same.

Linear equation	Literal equation	
$5(x+1) = 2x+7$	$5(x+y) = 2x+7y$	Original equation
$5x+5 = 2x+7$	$5x+5y = 2x+7y$	Simplify (distributive property)
$3x+5 = 7$	$3x+5y = 7y$	All x's in one member
$3x = 2$	$3x = 2y$	Terms not containing x in the other member
$x = \dfrac{2}{3}$	$x = \dfrac{2y}{3}$	Divide by the coefficient

In the linear equation, we have a solution for x, and in the literal equation, we have solved for x in terms of y.

Mastery points

Can you

- Solve literal equations and formulas for a specified variable?

Exercise 2–7

Solve for the specified variable. See example 2–7 A.

Example $P = 2\ell + 2w$, for ℓ

Solution

$P - 2w = 2\ell$ — Subtract $2w$

$\dfrac{P-2w}{2} = \ell$ — Divide by 2

$\ell = \dfrac{P-2w}{2}$ — Symmetric property

1. $V = \ell wh$, for w
2. $V = \ell wh$, for ℓ
3. $I = Prt$, for P
4. $I = Prt$, for t
5. $F = ma$, for m
6. $E = IR$, for R
7. $K = PV$, for V
8. $E = mc^2$, for m
9. $W = I^2R$, for R
10. $A = \ell w$, for w
11. $P = 2\ell + 2w$, for w
12. $C = \pi D$, for π
13. $P = a + b + c$, for a
14. $A = \dfrac{1}{2}bh$, for b
15. $ay - 3 = by + c$, for a
16. $ay - 3 = by + c$, for b
17. $V = k + gt$, for k
18. $V = k + gt$, for t
19. $A = \dfrac{1}{2}h(b + c)$, for b
20. $A = \dfrac{1}{2}h(b + c)$, for h
21. $\ell = a + (n - 1)d$, for a
22. $\ell = a + (n - 1)d$, for d
23. $A = P(1 + r)$, for P
24. $\ell = a + (n - 1)d$, for n
25. $T = 2f + g$, for f
26. $i = \dfrac{prm}{12}$, for r
27. $D = dq + R$, for q
28. $M = -P(\ell - x)$, for x
29. $R = W - b(2c + b)$, for c
30. $F = k\dfrac{m_1m_2}{d^2}$, for k
31. $A = P(1 + rt)$, for r
32. $A = P(1 + rt)$, for P
33. $V = r^2(a - b)$, for a
34. $P = n(P_2 - P_1) - c$, for P_2
35. $3x - y = 4x + 5y$, for x
36. $3x - y = 4x + 5y$, for y
37. $2S = 2vt - gt^2$, for g
38. $ax + by = c$, for y

39. The distance s that a body projected downward with an initial velocity of v will fall in t seconds because of the force of gravity is given by $s = \frac{1}{2}gt^2 + vt$. Solve for g.

40. Solve the formula in exercise 39 for v.

41. The net profit P on sales of n identical tape decks is given by $P = n(S - C) - e$, where S is the selling price, C is the cost to the dealer, and e is the operating expense. Solve for S.

42. Solve the formula in exercise 41 for C.

43. Solve the formula in exercise 41 for e.

Review exercises

Perform the indicated operations. See section 1–8.

1. -5^2
2. $(-5)^2$
3. -3^4
4. $(-3)^3$

Write an algebraic expression for each of the following. See section 2–1.

5. x raised to the fourth power
6. A number squared
7. The product of a and b
8. x multiplied by y

2–8 ■ Word problems

Many problems that we will encounter will be verbally stated. These will need to be translated into algebraic equations. In chapter 1, we solved arithmetic word problems. In section 2–1, we saw how to take a word phrase and write an algebraic expression for it.

We are now ready to combine our arithmetic problem-solving skills, our ability to write an algebraic expression, and our ability to solve an equation and apply them to solve word problems. On page 91, some useful guidelines for solving word problems were given. You should review them at this time.

■ Example 2–8 A

Write an equation for the problem and solve for the unknown quantities.

1. One number is 4 more than a second number. If their sum is 38, find the two numbers.

Note In problems where we are finding more than one value, it is usually easiest to let the unknown represent the smallest unknown value.

Let x = the smaller number (the second number). Then $x + 4$, which is 4 more than the smaller number, represents the other number. The parts that make up the equation are

smaller number	their sum	larger number	is	38
x	$+$	$(x + 4)$	$=$	38

$x + (x + 4) = 38$	Original equation
$x + x + 4 = 38$	Remove grouping symbol
$2x + 4 = 38$	Combine like terms
$2x = 34$	Subtract 4
$x = 17$	Divide by 2

Therefore the smaller number is 17 and the larger number is 4 more than the smaller number: $(x + 4)$ and $17 + 4 = 21$.

2. One number is 6 times a second number and their sum is 21. Find the numbers.

Let $x =$ the second number. Then six times the second number or $6x =$ the other number. The parts that make up the equation are

second number	their sum	other number	is	21
x	$+$	$6x$	$=$	21

$x + 6x = 21$ Original equation
$7x = 21$ Combine like terms
$x = 3$ Divide by 7

Hence the second number is 3 and the other number is 6 times the second number and is $6 \cdot (3) = 18$.

3. If the first of two consecutive integers is multiplied by 3, this product is 4 more than the sum of the two integers. Find the integers.

Note Prior knowledge that is needed for this problem is that consecutive integers differ by 1. Therefore we add 1 to the first to get the second, we would add 2 to the first to get a third, and so on.

first	second	third	fourth	fifth
x	$x + 1$	$x + 2$	$x + 3$	$x + 4$

first integer	second integer
x	$x + 1$

The parts that make up the equation are

three times the first integer	this product is	4	more than	the sum
$3x$	$=$	4	$+$	$[x + (x + 1)]$

$3x = 4 + [x + (x + 1)]$ Original equation
$3x = 4 + [x + x + 1]$ Remove innermost grouping symbol
$3x = 4 + [2x + 1]$ Combine like terms
$3x = 4 + 2x + 1$ Remove brackets
$3x = 2x + 5$ Combine like terms
$x = 5$ Subtract $2x$

Therefore the first consecutive integer is 5 and the second integer is one more than the first $(x + 1)$ and is $5 + 1 = 6$.

▶ ***Quick check*** One natural number is 5 times another natural number and their sum is 36. Find the numbers.

The sum of three consecutive integers is 36. Find the integers. ■

Mastery points

Can you

- Write an equation for a word problem?
- Solve for the unknown quantities?

Exercise 2–8

Write an equation for the problem and solve for the unknown quantities. See example 2–8 A.

Example One natural number is 5 times another natural number and their sum is 36. Find the numbers.

Solution Let x = the smaller number. Then five times the smaller number or $5x$ = the other number. The parts that make up the equation are

smaller number	sum	larger number	is	36
x	$+$	$5x$	$=$	36

$x + 5x = 36$ Original equation
$6x = 36$ Combine like terms
$x = 6$ Divide by 6

Therefore the smaller number is 6 and the larger number is 5 times the smaller number $(5x)$ and is $5 \cdot (6) = 30$.

1. One number is 18 more than a second number. If their sum is 62, find the two numbers.
2. One number is 9 less than another number. If their sum is 47, find the two numbers.
3. The difference of two numbers is 17. Find the numbers if their sum is 87.
4. If three times a number is increased by 11 and the result is 47, what is the number?
5. If a number is divided by 4 and that result is then increased by 6, the answer is 13. Find the number.
6. If a number is decreased by 14 and that result is then divided by 5, the answer is 15. Find the number.
7. Nine times a number is decreased by 4, leaving 59. What is the number?
8. One-third of a number is 8 less than one-half of the number. Find the number.
9. The difference between one-half of a number and one-third of the number is 9. Find the number.
10. One-half of a number minus one-third of the number is 8. Find the number.
11. What number added to its double gives 63?
12. Six times a number, increased by 10, gives 94. Find the number.
13. Find a number such that twice the sum of that number and 7 is 44.
14. One number is seven times another. If their difference is 18, what are the numbers?
15. Find two numbers whose sum is 63 and whose difference is 5.
16. One number is 11 more than twice a second number. If their sum is 35, what are the numbers?
17. One number is 9 times a second number and their sum is 120. Find the numbers.

Example The sum of three consecutive integers is 36. Find the integers.

Solution

first integer	second integer	third integer
x	$(x + 1)$	$(x + 2)$

The parts that make up the equation are

$$\underbrace{x + (x + 1) + (x + 2)}_{\text{the sum of three consecutive integers}} \underbrace{=}_{\text{is}} \underbrace{36}_{36}$$

$$\begin{aligned} x + (x + 1) + (x + 2) &= 36 && \text{Original equation} \\ x + x + 1 + x + 2 &= 36 && \text{Remove grouping symbols} \\ 3x + 3 &= 36 && \text{Combine like terms} \\ 3x &= 33 && \text{Subtract 3} \\ x &= 11 && \text{Divide by 3} \end{aligned}$$

Hence the first integer is $x = 11$, the second integer is $x + 1 = (11) + 1 = 12$, and the third integer is $x + 2 = (11) + 2 = 13$.

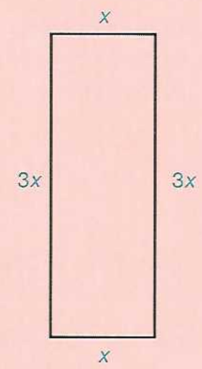
18. The sum of three consecutive even integers is 72. Find the integers.
19. The sum of three consecutive odd integers is 51. Find the integers.
20. One number is 4 times a second number and their sum is 65. Find the numbers.
21. One number is 7 times a second number and their sum is 96. Find the numbers.
22. The sum of three consecutive integers is 69. Find the integers.
23. The sum of three consecutive even integers is 66. Find the integers.
24. The sum of three consecutive odd integers is 75. Find the integers.
25. The sum of three consecutive integers is 93. Find the integers.
26. The sum of three numbers is 44. The second number is three times the first number and the third number is 6 less than the first number. Find the three numbers.
27. The sum of three numbers is 63. The first number is twice the second number and the third number is three times the first number. Find the three numbers.
28. One number is 7 more than another number. Find the two numbers if three times the larger number exceeds four times the smaller number by 5.
29. One number is 4 more than another number. Find the two numbers if two times the larger number is 7 less than five times the smaller number.
30. One number is 33 more than another. The smaller number is one-fourth of the larger number. Find the numbers.
31. A number plus one-half of the number plus one-third of the number equal 44. Find the number.
32. A number is decreased by 7 and twice this result is 52. What is the number?
33. Four times the first of three consecutive integers is 27 less than three times the sum of the second and third. Find the three integers.
34. Five times the first of three consecutive even integers is 2 less than twice the sum of the second and third. Find the three integers.
35. One-fourth of the middle integer of three consecutive even integers is 27 less than one-half of the sum of the other two integers. Find the three integers.

Example The length of a rectangle is 3 times its width and its perimeter is 40 feet. Find the dimensions.

Solution Let x = the width, then 3 times the width or $3x$ = the length.

$P = 2w + 2\ell$	Formula for perimeter
$40 = 2(x) + 2(3x)$	Substitute
$40 = 2x + 6x$	Multiply
$40 = 8x$	Combine like terms
$5 = x$	Divide by 8

Therefore the width is $x = 5$ feet and the length is $3x = 3(5) = 15$ feet.



36. The length of a rectangle is 9 feet more than its width. The perimeter of the rectangle is 58 feet. Find the dimensions. (Prior knowledge: Perimeter = 2 times the length plus 2 times the width.)

37. The width of a rectangle is 3 feet less than its length. The perimeter of the rectangle is 70 feet. Find the dimensions. (See exercise 36.)

38. The width of a rectangle is $\frac{1}{3}$ of its length. If the perimeter is 96 feet, find the dimensions.

39. The length of a rectangle is 1 inch less than three times the width. Find the dimensions if the perimeter is 70 inches.

40. The width of a rectangle is 3 meters less than the length. If the perimeter of the rectangle is 142 meters, find the dimensions of the rectangle.

41. The length of a rectangle is 5 feet more than its width. If the perimeter is 82 feet, find the length and width.

42. The sum of the number of teeth on two gears is 74 and their difference is 22. How may teeth are on each gear?

43. A 12-foot board is cut into two pieces so that one piece is 4 feet longer than the other. How long is each piece?

44. A 24-foot rope is cut into two pieces so that one piece is twice as long as the other. How long is each piece?

45. A 50-foot extension cord is cut into two pieces so that one piece is 12 feet longer than the other piece. How long is each piece?

46. The sum of two currents is 80 amperes. If the greater current is 24 amperes more than the lesser current, find their values.

47. Two gears have a total of 59 teeth. One gear has 15 less teeth than the other. How many teeth are on each gear?

48. Two electrical voltages have a total of 156 volts (V). If one voltage is 32 V more than the other, find the two voltages.

49. The sum of two voltages is 89 and their difference is 32. Find the two voltages.

50. The sum of two resistances in a series is 30 ohms and their difference is 14 ohms. How many ohms are in each resistor?

Example A man has \$10,000, part of which he invests at 11% and the rest at 8%. If his total income from the two investments for one year is \$980, how much does he invest at each rate?

Solution **Note** All interest problems in this textbook will be simple interest. The prior knowledge that is needed for this problem is that Interest = Principal · Rate · Time. Time will be equal to 1 year in the following problems.

11% investment	8% investment
x	$10{,}000 - x$

equation

$$x(0.11) + (10{,}000 - x)(0.08) = 980$$

If we have a total amount of \$10,000 to invest and we invest x dollars at 11%, then the amount left to invest at 8% would be the total amount minus what we have already invested, $10{,}000 - x$.

$$
\begin{aligned}
x(0.11) + (10{,}000 - x)(0.08) &= 980 && \text{Original equation}\\
0.11x + 800 - 0.08x &= 980 && \text{Distributive property}\\
0.03x + 800 &= 980 && \text{Combine like terms}\\
0.03x &= 180 && \text{Subtract 800}\\
x &= 6{,}000 && \text{Divide by 0.03}
\end{aligned}
$$

Hence he has invested $x = 6{,}000$ dollars at 11% and $10{,}000 - x = 10{,}000 - (6{,}000) = 4{,}000$ dollars at 8%.

51. Phil has \$20,000, part of which he invests at 8% interest and the rest at 6%. If his total income for one year was \$1,460 from the two investments, how much did he invest at each rate?

52. Nancy has \$18,000. She invests part of her money at 7½% interest and the rest at 9%. If her income for one year from the two investments was \$1,560, how much did she invest at each rate?

53. Tammy has \$15,000. She invest part of this money at 8% and the rest at 6%. Her income for one year from these investments totals \$1,120. How much is invested at each rate?

54. Alanzo invested \$26,000, part at 10% and the rest at 12%. If his income for one year from these investments is \$2,720, how much was invested at each rate?

55. Rich has \$18,000, part of which he invests at 10% interest and the rest at 8%. If his income from each investment was the same, how much did he invest at each rate?

56. Amy invests a total of \$12,000, part at 10% and part at 12%. Her total income for one year from the investments is \$1,340. How much is invested at each rate?

57. Barb has \$30,000, part of which she invests at 9% interest and the rest at 7%. If her income from the 7% investment was \$820 more than that from the 9% investment, how much did she invest at each rate?

58. Paul invested a total of \$18,000, part at 5% and part at 9%. If his income for one year from the 9% investment was \$200 less than his income from the 5% investment, how much was invested at each rate?

59. Lynne made two investments totaling \$25,000. On one investment she made an 18% profit, but on the other investment she took an 11% loss. If her net profit was \$2,180, how much was each investment?

60. Grace made two investments totaling \$18,000. She made a 14% profit on one investment, but she took a 9% loss on the other investment. If her net profit was \$220, how much was each investment?

61. Larry made two investments totaling \$21,000. One investment made him a 13% profit, but on the other investment, he took a 9% loss. If his net loss was \$196, how much was each investment?

62. Jeff made two investments totaling \$34,000. One investment made him a 12% profit, but on the other investment he took a 21% loss. If his net loss was \$2,940, how much was each investment?

63. Dale has invested \$5,000 at an 8% rate. How much more must he invest at 10% to make the total income for one year from both sources a 9% rate?

64. Jeremy has \$9,000 invested at 6%; how much more must he invest at 10% to realize a net return of 9%?

65. Jennifer has \$14,000 invested at 7% and is going to invest an additional amount at 11% so that her total investment will make 9%. How much does she need to invest at 11% to achieve this?

Review exercises

Evaluate the following formulas. See section 2–2.

1. $I = prt, p = 2{,}000; r = 0.05; t = 1$
2. $V = \ell wh, \ell = 7, w = 4, h = 3$
3. $F = ma, m = 34, a = 6$
4. $V = k + gt, k = 12, g = 32, t = 3$
5. $A = p + pr, p = 3{,}000; r = 0.06$
6. $A = \frac{1}{2}(b + c), b = 20, c = 12$
7. $S = \frac{1}{2}gt^2, g = 32, t = 4$
8. $\ell = a + (n - 1)d, a = 4, n = 10, d = 4$

2–9 ■ Solving linear inequalities

Inequality symbols

In chapter 1, we studied the meaning of the inequality symbols

$<$ "is less than"
$\leq$ "is less than or equal to"
$>$ "is greater than"
$\geq$ "is greater than or equal to."

These symbols define the *sense* or *order* of an inequality. Some examples of how we use these symbols would be:

1. If we want to state symbolically that 4 is less than 7, we write $4 < 7$.
2. If we wish to denote that the variable x represents 5 or any number greater than 5, we write $x \geq 5$.

Note $x \geq 5$ represents *any* real number that is greater than or equal to 5, and not just any integer greater than or equal to 5. Remember that 5.1, 5.004, and so on, are all greater than 5.

3. If we wish to denote that the variable T represents any number less than 3, but not 3 itself, we write $T < 3$.

Linear inequalities

When we replace the equal sign in a conditional linear equation with one of these inequality symbols, we form a *conditional linear inequality.*

$$2x \geq 6$$

Left member
Right member
Inequality symbol

A major difference between the linear equation and the linear inequality is the solution. The solution of a linear equation has at most one solution, whereas the solution of a linear inequality may consist of an unlimited number of solutions. Consider the inequality $2x \geq 6$. We can, by inspection, see that if we substitute 3, $3\frac{1}{2}$, 4, or 5 for x, the inequality will be true. In fact, we see that if

we were to substitute any number greater than or equal to 3, the inequality would be true. This demonstrates the fact that the inequality has an unlimited number of solutions. The values for x that would satisfy the inequality would be $x \geq 3$.

Another way to indicate the solution of an inequality is by graphing. To graph the solution, we simply draw a number line (as we did in chapter 1), place a solid circle at 3 on the number line to signify that 3 is in the solution, and draw an arrow extending from the solid circle to the right (figure 2–1). The solid line indicates that *all* numbers greater than or equal to 3 are part of the graph.

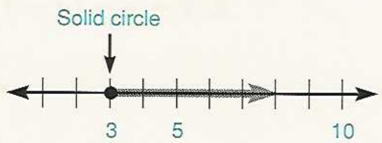
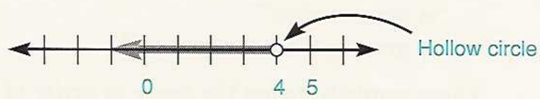


Figure 2–1

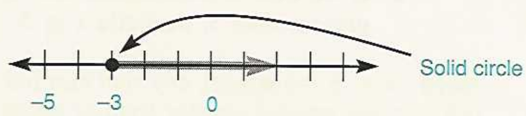
■ ***Example 2–9 A***

Graph the following linear inequalities.

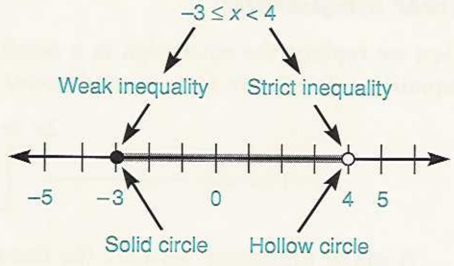
1. $x < 4$ Here x represents all real numbers less than 4, but not 4 itself. To denote the fact that x cannot equal 4, we put a **hollow circle** at 4.



2. $x \geq -3$ The greater than or equal to symbol, $\geq$, indicates that the graph will contain the point -3, and we place a **solid circle** at -3.



3. $-3 \leq x < 4$ This statement is called a *compound inequality.* It is read "-3 is less than or equal to x and x is less than 4." We place a solid circle at -3 to denote that -3 is included and place a hollow circle at 4 to show that 4 is not included. We then draw a line segment between the two circles.



Note When we graph inequalities, a strict inequality ($<$ or $>$) is represented by a hollow circle at the number. A weak inequality ($\leq$ or $\geq$) is represented by a solid circle at the number.

▶ ***Quick check*** Graph $-3 < x \leq 2$ ■

Solving linear inequalities

The properties that we will be using to solve linear inequalities are similar to those that we used to solve linear equations.

Addition and subtraction property of inequalities

For all real numbers a, b, and c, if $a < b$, then

$$a + c < b + c \text{ and } a - c < b - c$$

Concept

The same number can be added to or subtracted from both members of an inequality without changing the direction of the inequality symbol.

Multiplication and division property of inequalities

For all real numbers a, b, and c, if $a < b$, and

1. If $c > 0$ (c represents a positive number), then

$$a \cdot c < b \cdot c \text{ and } \frac{a}{c} < \frac{b}{c}$$

Concept

We can multiply or divide *both* members of the inequality by the same positive number without changing the direction of the inequality symbol.

2. If $c < 0$ (c represents a negative number), then

$$a \cdot c > b \cdot c \text{ and } \frac{a}{c} > \frac{b}{c}$$

Concept

We can multiply or divide *both* members of an inequality by the same *negative* number, provided that we **reverse** the direction of the inequality symbol.

Note The two properties were stated in terms of the less than ($<$) symbol. The properties apply for any of the other inequality symbols ($>$, $\leq$, or $\geq$).

To demonstrate these operations, consider the inequality $8 < 12$.

1. If we add or subtract 4 in each member, we still have a true statement.

$8 < 12$	or	$8 < 12$	Original true statement
$8 + 4 < 12 + 4$		$8 - 4 < 12 - 4$	Add or subtract 4
$12 < 16$		$4 < 8$	New true statement

2. If we multiply or divide by 4 in each member, we still have a true statement.

$8 < 12$	or	$8 < 12$	Original true statement
$8 \cdot 4 < 12 \cdot 4$		$\frac{8}{4} < \frac{12}{4}$	Multiply or divide by 4
$32 < 48$		$2 < 3$	New true statement

3. But if we multiply or divide by -4 in each member, we have to reverse the direction of the inequality before we have a true statement.

$8 < 12$	or	$8 < 12$	Orignial true statement
$8(-4) > 12(-4)$		$\dfrac{8}{-4} > \dfrac{12}{-4}$	Multiply or divide by -4 and reverse direction of the inequality symbol
$-32 > -48$		$-2 > -3$	New true statement

Note When we reverse the direction of the inequality symbol, we say that we **reversed the sense or order** of the inequality.

To summarize our properties, we see that they are the same as the properties for linear equations, with one exception. **Whenever we multiply or divide both members of an inequality by a negative number, we must reverse the direction of the inequality symbol.**

We shall now solve some linear inequalities. The procedure for solving a linear inequality uses the same four steps that we used to solve a linear equation.

Solving a linear inequality

1. Simplify in each member, where necessary, by performing the indicated operations.
2. Add, or subtract, to get all terms containing the unknown in one member of the inequality.
3. Add, or subtract, to get all terms *not* containing the unknown in the other member of the inequality.
4. Multiply, or divide, to obtain a coefficient of 1 for the unknown. *Remember, when multiplying or dividing by a negative number, always change the direction (order) of the inequality symbol.*

■ *Example 2–9 B*

Find the solution.

1. $2x + 5x - 1 < 4x + 2$

Step 1 *We simplify* the inequality by carrying out the indicated addition in the left member.

$$2x + 5x - 1 < 4x + 2$$
$$7x - 1 < 4x + 2$$

Step 2 *We want all the terms containing the unknown,* x, *in one member of the inequality.* Therefore we subtract $4x$ from both members of the inequality.

$$7x - 1 < 4x + 2$$
$$7x - 4x - 1 < 4x - 4x + 2$$
$$3x - 1 < 2$$

Note A negative coefficient of the unknown can be avoided if we form equivalent inequalities where the unknown appears only in the member of the inequality that has the greater coefficient of the unknown.

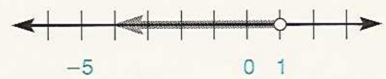
Step 3 *We want all the terms not involving the unknown in the other member of the inequality.* Therefore we add 1 to both members of the inequality.

$$3x - 1 < 2$$
$$3x - 1 + 1 < 2 + 1$$
$$3x < 3$$

Step 4 *We form an equivalent inequality where the coefficient of the unknown is 1.* Hence we divide both members of the inequality by 3.

$$3x < 3$$
$$\frac{3x}{3} < \frac{3}{3}$$
$$x < 1$$

We can also graph the solution.



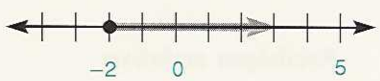
Note We should be careful to observe in step 4 whether we are multiplying or dividing by a positive or negative number so that we will form the correct inequality.

2. $-2x \leq 4$

The only operation we need to perform to solve the inequality is to divide by -2. Since we are dividing by a negative number, we must remember to **reverse** the direction of the inequality symbol.

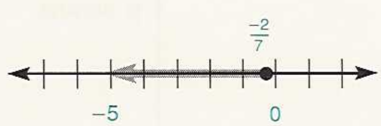
$$-2x \leq 4$$
$$\frac{-2x}{-2} \geq \frac{4}{-2} \qquad \text{Reverse the direction of the inequality symbol}$$
$$x \geq -2$$

Graph



3.
$$5(2x + 1) \leq 7x - 4x + 3$$
$$10x + 5 \leq 3x + 3 \qquad \text{Simplify by multiplying in left member and combining like terms in right member}$$
$$10x - 3x + 5 \leq 3x - 3x + 3 \qquad \text{Subtract } 3x \text{ from both members}$$
$$7x + 5 \leq 3$$
$$7x + 5 - 5 \leq 3 - 5 \qquad \text{Subtract 5 from both members}$$
$$7x \leq -2$$
$$\frac{7x}{7} \leq \frac{-2}{7} \qquad \text{Divide both members by 7}$$
$$x \leq \frac{-2}{7}$$

Graph



4. $-3 \leq 2x + 1 < 5$

When solving a compound inequality, the solution must be such that the unknown appears only in the middle member of the inequality. We can still use all of our properties, if we apply them to all three members, and we must reverse the direction of *all* inequality symbols when multiplying or dividing by a negative number.

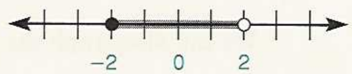
$$-3 \leq 2x + 1 < 5$$

$$-3 - 1 \leq 2x + 1 - 1 < 5 - 1 \quad \text{Subtract 1 from all three members}$$

$$-4 \leq 2x < 4$$

$$\frac{-4}{2} \leq \frac{2x}{2} < \frac{4}{2} \quad \text{Divide all three members by 2}$$

$$-2 \leq x < 2$$

Graph 

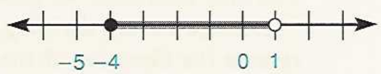
5. $-6 < -3x - 3 \leq 9$

$$-6 + 3 < -3x - 3 + 3 \leq 9 + 3 \quad \text{Add 3 to all three members}$$

$$-3 < -3x \leq 12$$

$$\frac{-3}{-3} > \frac{-3x}{-3} \geq \frac{12}{-3} \quad \text{Divide all three members by } -3 \text{, reversing the direction of both inequality symbols}$$

$$1 > x \geq -4$$

Graph 

Note From our discussions in chapter 1, we could have written the solution in the previous problem as $-4 \leq x < 1$. This is usually the preferred form.

▶ ***Quick check*** Find the solution for $4x + 5x - 4 < 6x - 1$ ■

Problem solving

We are now ready to combine our abilities to write an expression and to solve an inequality and apply them to solve word problems. The guidelines for solving a linear inequality are the same as those for solving a linear equation in section 2–4. The following table shows a number of different ways that an inequality symbol could be written with words.

Symbol	$<$	$\leq$	$>$	$\geq$
In words	is less than is fewer than is almost	is at most is no more than is no greater than is less than or equal to	is greater than is more than exceeds	is at least is no less than is no fewer than is greater than or equal to

■ Example 2–9 C

1. Write an inequality for the following statement: A student's test grade, G, must be at least 75 to have a passing grade.

If the student's test grade must be *at least* 75, the grade must be 75 or greater. Thus,

$G \geq 75$

2. Four times a number less 5 is to be no more than three times the number increased by 2. Find the number.

Let x represent the number.

4 times a number	less	5	is no more than	3 times the number	increased by	2
$4x$	$-$	5	$\leq$	$3x$	$+$	2

The inequality is $4x - 5 \leq 3x + 2$

$$
\begin{aligned}
4x - 5 &\leq 3x + 2 \\
4x - 5 - 3x &\leq 3x + 2 - 3x && \text{Subtract } 3x \text{ from each member} \\
x - 5 &\leq 2 && \text{Combine in each member} \\
x - 5 + 5 &\leq 2 + 5 && \text{Add 5 to each member} \\
x &\leq 7 && \text{Combine in each member}
\end{aligned}
$$

The number is any real number x such that $x \leq 7$.

▶ ***Quick check*** To complete an order for cement, a company will need at least 3 trucks. Write an inequality for the number of trucks needed.

If 4 is subtracted from three times a number, the result is greater than 2 more than twice the number. Find the number. ■

Mastery points

Can you

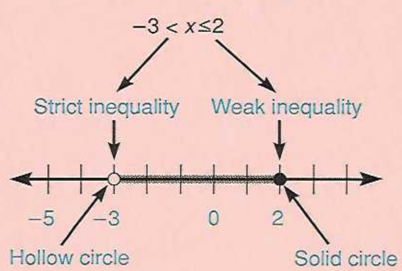
- Graph inequalities and compound inequalities?
- Solve linear inequalities and compound linear inequalities?
- Solve verbal inequalities?

Exercise 2–9

Graph the following. See example 2–9 A.

Example $-3 < x \leq 2$

Solution



1. $x > 2$ **2.** $x > -2$ **3.** $x \geq 1$ **4.** $x \geq 4$ **5.** $x < 0$

6. $x < -2$ **7.** $x > 0$ **8.** $x \leq 3$ **9.** $x \leq -4$ **10.** $-2 < x < 0$

11. $-1 < x < 2$ **12.** $-3 \leq x \leq 4$ **13.** $0 \leq x \leq 5$ **14.** $1 \leq x < 4$ **15.** $-1 < x \leq 3$

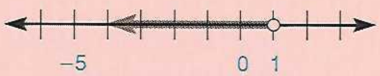
Find the solution and graph the solution. See example 2–9 B.

Example $4x + 5x - 4 < 6x - 1$

Solution

$9x - 4 < 6x - 1$	Combine like terms in left member
$9x - 4 - 6x < 6x - 1 - 6x$	Subtract $6x$ from each member
$3x - 4 < -1$	Combine like terms
$3x - 4 + 4 < -1 + 4$	Add 4 to each member
$3x < 3$	Combine like terms
$x < 1$	Divide each member by 3

Thus $x < 1$ and the solutions are all real numbers less than 1.



16. $4x > 10$ **17.** $2x \leq 5$ **18.** $3x \geq 15$

19. $5x < 30$ **20.** $\frac{3}{4}x < 9$ **21.** $\frac{2}{3}x \geq 12$

22. $-4x < 12$ **23.** $-3x \leq 27$ **24.** $-6x > 18$

25. $-2x < 10$ **26.** $4x + 3x \geq 2x + 7$ **27.** $8x - 2x > 4x - 5$

28. $3x + 2x < x + 6$ **29.** $\frac{3x}{2} > 8$ **30.** $\frac{4x}{3} > 12$

31. $x + 7 \leq 9$ **32.** $x + 4 \geq -12$ **33.** $x - 5 < 6$

34. $x - 12 < -9$ **35.** $2x + (3x - 1) > 5 - x$ **36.** $2(3x + 1) < 7$

37. $3(2x - 1) \geq 4x + 3$ **38.** $4(5x - 3) \leq 25x + 11$ **39.** $12x - 8 > 5x + 2$

40. $9x + 4 > x - 11$ **41.** $2 + 5x - 16 < 6x - 4$ **42.** $3(2x + 5) > 4(x - 3)$

43. $8 - 2(3x + 4) > 5x - 16$ **44.** $(3x + 2) - (2x - 5) > 7$ **45.** $(7x - 6) - (4 - 3x) \leq 27$

46. $2(x - 4) - 16 \leq 3(5 - 2x)$ **47.** $3(1 - 2x) \geq 2(4 - 4x)$ **48.** $4(5 - x) > 7(2 - x)$

49. $3(2x + 3) \geq 5 - 4(x - 2)$ **50.** $6(3x - 2) \leq 7(x - 3) - 2$ **51.** $-1 < 2x + 3 < 4$

52. $-3 < 3x - 4 < 6$ **53.** $-2 \leq 5x + 2 \leq 3$ **54.** $0 \leq 7x - 1 \leq 7$

55. $-5 < 4x + 3 \leq 8$ **56.** $-2 < -x \leq 3$ **57.** $-1 \leq -x < 4$

58. $-4 \leq 2 - x < 3$ **59.** $-3 < 4 - x \leq 5$ **60.** $1 < 3 - 4x < 6$

61. $0 \leq 1 - 3x < 7$ **62.** $-4 \leq 3 - 2x \leq 0$

Write an inequality to represent the following statements. See Example 2–9 C.

Example To complete an order for cement, a company will need at least 3 trucks.

Solution The words *at least* 3 trucks means that the company will need *3 or more* trucks. If x is the number of trucks needed, then

$x \geq 3$

63. Mark's score must be at least 72 on the final exam to pass the course.
64. The temperature today will be less than 38.
65. An automobile parts company needs to order at least 8 new lift trucks.
66. An accounting company will hire at least 2 new employees, but not more than 7.
67. The selling price (P) must be at least twice the cost (C).

Write an inequality using the given information and solve. See example 2–9 C.

Example If 4 is subtracted from three times a number, the result is greater than 2 more than twice the number. Find the number.

Solution Let x represent the number. Then

$3x - 4$ is "4 subtracted from three times the number,"
$2x + 2$ is "2 more than twice the number."

Since the two expressions are related by "is greater than," the inequality is $3x - 4 > 2x + 2$

$$3x - 4 > 2x + 2$$
$$3x - 4 - 2x > 2x + 2 - 2x \quad \text{Subtract } 2x \text{ from each member}$$
$$x - 4 > 2 \quad \text{Combine in each member}$$
$$x - 4 + 4 > 2 + 4 \quad \text{Add 4 to each member}$$
$$x > 6 \quad \text{Combine in each member}$$

The number is any number that is greater than 6.

68. When 7 is subtracted from two times a number, the result is greater than or equal to 9. Find all numbers that satisfy this condition.
69. Five times a number minus 11 is less than 19. Find all numbers that satisfy this condition.
70. The product of 6 times a number added to 2 is greater than or equal to 1 subtracted from five times the number. What are the numbers that satisfy this condition?
71. Twice a number increased by 7 is no more than three times the number decreased by 5. Find the numbers that satisfy this condition.
72. If one-third of a number is added to 23, the result is greater than 30. Find all numbers that satisfy this condition.
73. Eugenia has scores of 7, 6, and 8 on three quizzes. What must she score on the fourth quiz to have an average of 7 or higher?
74. Sam has scores of 72, 67, and 81 on three tests. If an average of 70 is required to pass the course, what is the minimum score he must have on the fourth test to pass?
75. Two times a number plus 4 is greater than 6 but less than 14. Find all numbers that satisfy these conditions.
76. Four times a number minus 7 is greater than 17 but less than 25. Find all numbers that satisfy these conditions.
77. The perimeter of a square must be greater than 20 inches but less than 108 inches. Find all values of a side that satisfy these conditions.
78. The perimeter of a rectangle must be less than 100 feet. If the length is known to be 30 feet, find all numbers that the width could be. (*Note:* The width of a rectangle must be a positive number.)

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79. The perimeter (the sum of the sides) of a triangle is more than 52 cm. If two sides of the triangle are 18 cm and 16 cm, respectively, what are the possible values for the length of the third side?

80. Two sides of a triangle are 10 ft and 12 ft long, respectively. If the perimeter must be at least 31 ft, what are the possible values for the length of the third side?

Review exercises

Perform the indicated operations. See section 1–8.

1. -4^2
2. $(-4)^2$
3. -2^4
4. $(-2)^4$

Write an algebraic expression for each of the following. See section 2–1.

5. x raised to the fifth power
6. A number cubed
7. A number squared
8. The product of x and y

Chapter 2 lead-in problem

Bonnie has $3,000 invested at 8% simple interest per year. How much more money must she invest at 7% simple interest if she wants an income of $660 per year ($55 per month) from her investments?

Solution

Let x = the number of dollars invested at 7%.

$$\underbrace{3{,}000(0.08)}_{\text{income from 8\% investment}} + \underbrace{x(0.07)}_{\text{income from 7\% investment}} = \underbrace{660}_{\text{total income}} \quad \text{Original equation}$$

$$240 + 0.07x = 660 \quad \text{Simplify}$$

$$0.07x = 420 \quad \text{Subtract 240}$$

$$x = 6{,}000 \quad \text{Divide by 0.07}$$

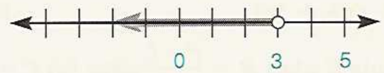
Therefore Bonnie needs to invest $6,000 at 7% so that her total income from both investments is $660 per year.

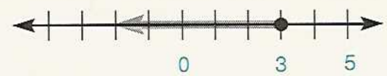
Chapter 2 summary

1. A **variable** is a symbol (generally a lowercase letter) that represents an unspecified number.
2. A **constant** is a symbol that does not change its value.
3. An **algebraic expression** is any meaningful collection of variables, constants, grouping symbols, and signs of operations.
4. The **terms** in an algebraic expression are any constants, variables, or products or quotients of these. They are separated by plus or minus signs.
5. In the expression $8x$, 8 is called the **numerical coefficient** or just the coefficient.
6. A **polynomial** is a special kind of algebraic expression. A **monomial** is a polynomial that contains one term; a **binomial** contains two terms; a **trinomial** contains three terms; a **multinomial** contains more than one term.
7. We use the *property of substitution* to *evaluate* algebraic expressions.
8. **Like terms** or **similar terms** are terms whose variable factors are the same.
9. We can *add or subtract* only like, or similar, terms.
10. A **mathematical statement** can be labeled true or false.
11. An equation that is true for every permissible value of the variable is called an **identity.**
12. A replacement value for the variable that forms a true statement (satisfies that equation) is called a **root,** or a **solution,** of the equation.
13. The **solution set** is the set of all values for the variable that cause the equation to be a true statement.
14. A **linear equation** is an equation where the exponent of the unknown is 1.
15. The **addition and subtraction property of equality** enables us to add or subtract the same quantity in each member of an equation and the result will be an equivalent equation.
16. The **symmetric property of equality** allows us to interchange right and left members of an equation.

17. The **multiplication and division property of equality** enables us to multiply or divide both members of an equation by the same nonzero quantity.
18. We use the same procedures for solving **literal equations** that we use to solve linear equations in one variable.
19. A linear inequality involves the symbols $<$, $\leq$, $>$, and $\geq$.
20. The **addition and subtraction property of inequalities** states that the same number can be added to or subtracted from both members of an inequality without changing the direction (order) of the inequality symbol.
21. The **multiplication and division property of inequalities** states:
    a. The same *positive* number may be multiplied times or divided into both members of an inequality without changing the direction (order) of the inequality.
    b. When the same *negative* number is multiplied times or divided into both members of an inequality, the direction (order) of the inequality *must be changed.*

Chapter 2 error analysis

1. Degree of a polynomial
   Example: $2x - 3x^2 + x^3 - 1$ has degree 6.
   Correct answer: $2x - 3x^2 + x^3 - 1$ has degree 3.
   What error was made? (*see page 69*)
2. Terms in an algebraic expression
   Example: $x^2 + \dfrac{2x - 3}{5}$ has 3 terms.
   Correct answer: $x^2 + \dfrac{2x - 3}{5}$ has 2 terms.
   What error was made? (*see page 68*)
3. Applying the distributive property
   Example: $5(4 + b) = 20b$
   Correct answer: $5(4 + b) = 20 + 5b$
   What error was made? (*see page 80*)
4. Combining like terms
   Example: $3a^2 + 4a = 7a^3$
   Correct answer: $3a^2 + 4a = 3a^2 + 4a$
   What error was made? (*see page 81*)
5. Combining polynomials
   Example: $(3x^2 - 2x + 1) - (x^2 - x + 2)$
   $= 3x^2 - 2x + 1 - x^2 - x + 2$
   $= 2x^2 - 3x + 3$
   Correct answer: $(3x^2 - 2x + 1) - (x^2 - x + 2)$
   $= 2x^2 - x - 1$
   What error was made? (*see page 82*)
6. Reciprocal of a number
   Example: The reciprocal of 0 is $\dfrac{1}{0}$.
   Correct answer: 0 has no reciprocal.
   What error was made? (*see page 94*)
7. Graphing linear inequalities
   Example: The graph of $x \leq 3$ is
   

   Correct answer: The graph of $x \leq 3$ is
   

   What error was made? (*see page 114*)
8. Multiplying members of an inequality
   Example: If $3 < 4$, then $3 \cdot -2 < 4 \cdot -2$.
   Correct answer: If $3 < 4$, then $3 \cdot -2 > 4 \cdot -2$.
   What error was made? (*see page 115*)
9. Multiplication of negative numbers
   Example: $(-5)(7) = 35$
   Correct answer: $(-5)(7) = -35$
   What error was made? (*see page 48*)
10. Division using zero
    Example: $\dfrac{7}{0} = 0$
    Correct answer: $\dfrac{7}{0}$ is undefined.
    What error was made? (*see page 53*)

Chapter 2 critical thinking

If you add any three consecutive odd integers, the sum will be a multiple of 3. Why is this true?

Chapter 2 review

[2–1]

Specify the number of terms in each expression.

1. $4x^2 + 3x + 2$
2. $5a^2b$
3. $7xy + 5$
4. $(ab + cd) + xy$

Determine which of the following algebraic expressions are polynomials. If they are not polynomials, state why not.

5. $\frac{x + y}{3} + z$
6. $x^3 - x^2$
7. $4a^2b^3c$
8. $\frac{a + b}{c}$

Write an algebraic expression for each of the following.

9. 5 times x
10. 7 less than y
11. 4 more than z
12. 2 times a number, plus 6

[2–2]

Evaluate the following expressions if $a = 3$, $b = 4$, $c = -4$, and $d = -3$.

13. $3a - b + c$
14. $d - 2(a + c)$
15. $a^2b - a^2c$
16. $(2a + c)(b + 2d)$
17. $(c - 2d)^2$
18. $c^2 - d^2$
19. Evaluate R when $R = \frac{P \cdot L}{D^2}$ given (a) $P = 6$, $L = 8$, $D = 4$; (b) $P = 7$, $L = 3$, $D = \frac{2}{3}$.
20. The volume of a gas V_2 is given by $V_2 = \frac{P_1V_1}{P_2}$. Find V_2 when $P_1 = 780$, $V_1 = 80$, and $P_2 = 60$.

[2–3]

Remove all grouping symbols and combine like terms.

21. $(3x^2 + 2x - 1) + (x^2 - 5x + 4)$
22. $(a^2 - 3a + 4) - (2a^2 - 4a - 7)$
23. $(4a^2 - b^2) - (3a^2 + 2b^2) - (7a^2 - 3b^2)$
24. $(5x^3 - 2xy^2 + 3x^2y - 4y^3) - (4x^2 + 3x^2y - 2y^3 + 5xy^2)$
25. $(4ab + 7b^2c) - (15ab - 11bc)$
26. $(x - 2y + 7) - (x + 4y + 6)$
27. $(4ab - 2ac) - (6bc - 5ac) + (ab + 2bc)$
28. $3a - [4a - (a - 5)]$
29. $5x + [3x - (x - y)]$
30. $4x - (x - y) - [3x - y - (2x + 3y)]$
31. $x - \{5x - [3y - (2x - y)]\}$
32. $5a - \{6b + a - (5a - 4b)\}$

[2–4]

Determine whether the given statement is true or false when we replace the variable in each equation with the given number.

33. $x + 7 = 11; \{4\}$
34. $2x + 1 = 9; \{2\}$
35. $5x - 1 = 21; \{5\}$
36. $\frac{x}{2} + 5 = 12; \{14\}$

[2–4, 2–5, 2–6]

Find the solution set.

37. $x + 5 = 12$
38. $x - 4 = 17$
39. $a + 7 = -4$
40. $b - 3 = -9$
41. $5z + 3z - 7z + 3 = 7$
42. $2(3x - 4) - 5x = 11$
43. $3(2y + 3) = 7 + 5y$
44. $3(x - 1) - 2(x + 1) = 4$
45. $3x = 9$
46. $4x = 12$
47. $-2x = 14$
48. $-3x = 21$
49. $\frac{x}{3} = 4$
50. $\frac{x}{2} = 7$
51. $\frac{3x}{5} = 9$
52. $\frac{2x}{7} = 6$
53. $\frac{1}{3}x - 1 = \frac{3}{4}$
54. $\frac{1}{3}x + 1 = \frac{1}{6}x - 2$
55. $\frac{3}{4}x + 4 = \frac{5}{8}$
56. $\frac{3}{5}x + \frac{1}{2} = \frac{7}{10}x - 3$
57. $3x = 0$

58. $-x = -4$
59. $3.7a = 22.2$
60. $32.8 = -4.1x$
61. $2x + 5 = 11$
62. $3b - 8 = 6$
63. $y + (2y - 1) = 6$
64. $x + 3x = 5 + 7$
65. $3(2a - 1) = 4a - 2$
66. $5(x + 3) = 2x - 7$
67. $(3x - 2) - (4x - 1) = 3x$
68. $2a + 5a - 4 = 3(1 - 2a)$
69. $8 - 3x + 7 = 5(x + 7)$
70. $2y - 3(y + 1) = 11$
71. $7x - 4(2x + 3) = 12$
72. $8x - 14 = 14 - 8x$
73. $5b + 4 = 4 - 2b$
74. $-3(2x + 1) = 4x - 5$
75. $3(c + 2) - 2(c + 1) = 5c + 11$
76. $4x - 2(1 - 3x) = 8x + 2$

[2–7]

Solve for the specified variable.

77. $F = ma$, for a
78. $E = IR$, for I
79. $k = PV$, for P
80. $V = k + g + t$, for g
81. $A = \frac{1}{2}h(b + c)$, for c
82. $5x - y = 2x + 3y$, for x

[2–8]

Write an equation for the problem and solve for the unknown quantities.

83. The difference between two numbers is 23. Find the two numbers if their sum is 105.
84. If a number is divided by 9 and that result is then increased by 7, the answer is 11. Find the number.
85. The difference between one-third of a number and one-fifth of a number is 6. Find the number.
86. John invested part of $20,000 at 8% and the rest at 7%. If his income from the 8% investment was $250 more than that from the 7% investment, how much was invested at each rate?
87. Anne made two investments totaling $25,000. On one investment she made a 12% profit but on the other she took a 19% loss. If her net loss was $1,030, how much was in each investment?

[2–9]

Find the solution and graph the solution.

88. $3x > 12$
89. $5x \leq 15$
90. $-2x < 14$
91. $-4x > 16$
92. $2x + 1 < 5$
93. $7x - 4 > 11$
94. $3x + 7 < 5x - 2$
95. $9x + 13 \geq 4x + 7$
96. $6(2x - 1) \leq 3x - 4$
97. $-4 < 5x + 7 < 10$
98. $0 \leq 1 - 5x < 6$
99. $5 < 4x + 3 < 12$
100. $-8 \leq 3x + 5 \leq 4$

Chapter 2 cumulative test

Perform the indicated operations, if possible, and simplify.

[1–4] 1. $(-8) + (-4)$
[1–5] 2. $(-10) - (-14)$
[1–7] 3. $\frac{-24}{-8}$
[1–7] 4. $\frac{8}{0}$
[1–8] 5. -5^2
[1–2] 6. $\frac{4}{5} - \frac{3}{10}$
[1–4] 7. $\frac{2}{3} + \left(-\frac{5}{6}\right)$
[1–6] 8. $(-4)(0)(-2)$
[1–2] 9. $(2.3)(8.6)$
[1–8] 10. $5 + 6(8 - 2)$
[1–8] 11. $6 + 4(10 - 2)$
[1–8] 12. $10 - 2(15 - 3) - 5 \cdot 2$
[1–8] 13. $2[5(7 - 4) - 6 + 4]$
[1–8] 14. $5(-4 + 7) - 3(8 - 5)$
[1–8] 15. $14 + 2 \cdot 15 \div 6 - 3 + 4$
[2–3] 16. $5x + x - 2x$
[2–3] 17. $3x^2y^2 - 2xy - x^2y^2 + 5xy$
[2–3] 18. $(2a - b) - (a - 4b)$
[2–3] 19. $(3x^2y - 2xy^2) - (5xy^2 - x^2y)$
[2–3] 20. $5a + 3a^2 - 4a - a^2 + 5 + a^3 - 6$
[2–3] 21. $x - [3x - (y + x) + (2x - 3y)]$
[2–3] 22. $(3a - 2b) - [5a - (4b + 6a)]$

Evaluate the following if $a = -2$, $b = -3$, $c = 4$, and $d = 5$.

[2–2] **23.** $(a + 2d)^2$

[2–2] **24.** $(3a - 2b) - (5c + d)$

[2–2] **25.** $(a - 4c)(b - 2d)$

Find the answer.

[1–4] **26.** The sum of 8, -12, and 6

[1–5] **27.** Subtract -12 from -8.

[1–6] **28.** There are 6 rows of desks in a classroom. If each row contains 7 desks, how many desks are in the classroom?

[1–7] **29.** A trip of 357 miles takes seven hours to complete. What was the average rate of speed?

Write an algebraic expression for each of the following.

[2–1] **30.** x decreased by y

[2–1] **31.** A number increased by 6

[2–2] **32.** Ann has d dimes, n nickels, and c cents. Express in cents the amount of money Ann has.

Find the solution set for 33–37 and the solution for 38–42.

[2–6] **33.** $10x - 7 = 4x + 3$

[2–6] **34.** $5x + 6 = 6$

[2–6] **35.** $\frac{1}{3}x + 4 = \frac{5}{6}$

[2–6] **36.** $3(2x - 1) + 2(5x - 3) = 8$

[2–6] **37.** $16 - 2(4x - 1) = 3x - 12$

[2–9] **38.** $-2x \geq 12$

[2–9] **39.** $5x + 3x < 6x - 14$

[2–9] **40.** $3x + (x - 1) > 7 - x$

[2–9] **41.** $-1 < 2x + 3 < 11$

[2–9] **42.** $-16 \leq 8 - 4x \leq 12$

Solve for the specified variable.

[2–7] **43.** $P = a + b + c$, for b

[2–7] **44.** $x = a(y + z)$, for y

Solve the following word problems.

[2–8] **45.** Phil has \$10,000, part of which he invests at 6% and the rest at 5%. If his total income from the two investments was \$560, how much did he invest at each rate?

[2–8] **46.** The sum of three consecutive even integers is 48. Find the three integers.

[2–8] **47.** If a number is increased by 9 and that result is divided by 3, the answer is 7. Find the number.

[2–8] **48.** Dwala made two investments totaling \$17,000. On one investment she made a 12% profit but on the other she took a 19% loss. If her net loss was \$1,215, how much was in each investment?

[2–9] **49.** Twice a number decreased by 2 is at most 10. Find all numbers that satisfy this condition.

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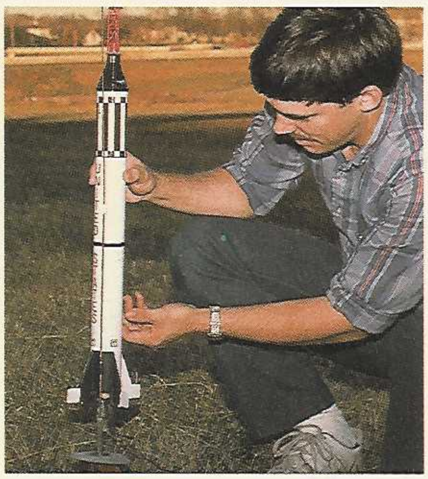
Chapter 3 ■ Polynomials and exponents

Chapter 4 ■ Factoring and solution of quadratic equations by factoring



Chapter 5 ■ Rational Expressions, Ratio and Proportion

Chapter 6 ■ Operations with Rational Expressions

Chapter 7 ■ Linear Equations in Two Variables

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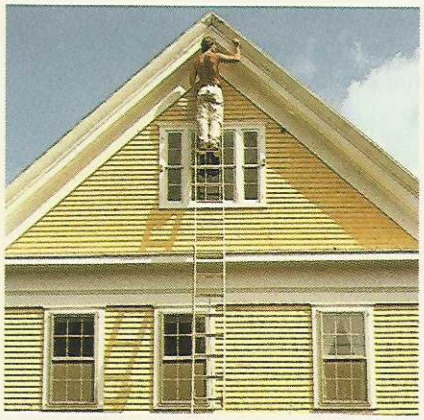
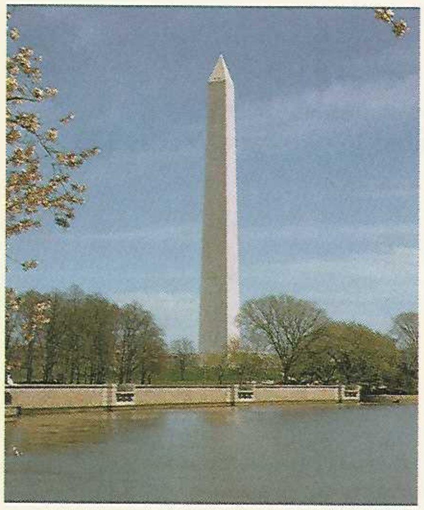
Chapter 9 ■ Roots and Radicals



Chapter 10 ■ Solutions of Quadratic Equations



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54. $\left(\frac{6-3}{7-4}\right)\left(\frac{14+2\cdot 3}{5}\right) = \left(\frac{3}{3}\right)\left(\frac{14+6}{5}\right) = \left[\frac{3}{3}\right]\left[\frac{20}{5}\right]$
$= 1 \cdot 4 = 4$ **59.** $\frac{22}{7}\cdot 3^2 - \frac{22}{7}\cdot 2^2 = \frac{22}{7}\cdot 9 - \frac{22}{7}\cdot 4$
$= \frac{198}{7} - \frac{88}{7} = \frac{198-88}{7} = \frac{110}{7}$ or $15\frac{5}{7}$ square inches

65. Let p = the total number of pieces of lumber. *Dividing* the 16-foot board by 4 and the 12-foot board by 3 will give us the number of pieces of lumber. If we *add* the number of pieces from the 16-foot board to the number of pieces from the 12-foot board, we will have the total number of pieces.

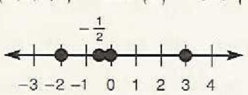
total number of pieces	is	number of pieces from the 16-foot board	combined with	number of pieces from the 12-foot board
p	$=$	$16 \div 4$	$+$	$12 \div 3$

$p = 16 \div 4 + 12 \div 3$
$= 4 + 4$ Priority 3
$= 8$ Priority 4

There will be 8 pieces of lumber.

Chapter 1 review

1. $\frac{5}{7}$ **2.** $\frac{3}{4}$ **3.** $\frac{2}{3}$ **4.** $\frac{10}{7}$ or $1\frac{3}{7}$ **5.** $\frac{3}{5}$ **6.** $\frac{21}{20}$ or $1\frac{1}{20}$
7. $\frac{7}{8}$ **8.** $\frac{25}{8}$ or $3\frac{1}{8}$ **9.** $\frac{25}{3}$ or $8\frac{1}{3}$ **10.** $\frac{5}{8}$ acre **11.** $\frac{2}{5}$ cup
12. $\frac{8}{7}$ or $1\frac{1}{7}$ **13.** $\frac{19}{24}$ **14.** $\frac{5}{6}$ **15.** $\frac{2}{9}$ **16.** $6\frac{17}{20}$ **17.** $\frac{2}{15}$
18. $\frac{7}{12}$ **19.** $\frac{7}{8}$ acre **20.** 263.51 **21.** 31.795 **22.** 1,355.09
23. 14.3 **24.** \$565.49 **25.** 7.86 acres **26.** $\approx$ 12.42 mpg
27. 10 **28.** 68.4 **29.** 25 **30.** 78.72 **31.** $\{50,51,52,53,54,55\}$
32. $\{1,2,3,4\}$ **33.** $\{0\}$ **34.** $\{-3,-2,-1,0,1,2,3\}$

35.


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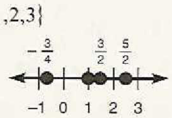

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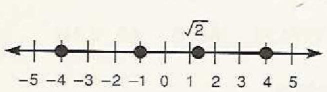

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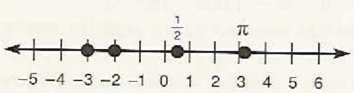


39. $<$ **40.** $<$ **41.** $>$ **42.** $<$ **43.** $>$ **44.** $>$ **45.** -4
46. 3 **47.** -6 **48.** 1 **49.** -6 **50.** 15 **51.** -4
52. -9 **53.** 3 **54.** 15 **55.** -21 **56.** 12 **57.** 24
58. -144 **59.** 0 **60.** -7 **61.** 2 **62.** -6 **63.** undefined
64. 0 **65.** indeterminate **66.** -1 **67.** $-9, -4$
68. **a.** $52{,}000 + (-3{,}000) + (-2{,}560) + (-3{,}300)$ **b.** \$43,140
69. 40° **70.** **a.** $+9, +8, -5, -6$ **b.** $>, +6$ **c.** 58°, $69 + (-11) = 58$ **71.** 25 **72.** -64 **73.** -16 **74.** -27
75. 98 **76.** -3 **77.** 20 **78.** 49 **79.** -9 **80.** 20
81. -14 **82.** 27

Chapter 2

Exercise 2–1

Answers to odd-numbered problems

1. 2 terms **3.** 3 terms **5.** 1 term **7.** 3 terms **9.** 2 terms **11.** 1 term **13.** 2 terms **15.** 5 is the coefficient of x^2, 1 is understood to be the coefficient of x, -4 is the coefficient of z **17.** 1 is understood to be the coefficient of x, -1 is understood to be the coefficient of y, -3 is the coefficient of z **19.** -2 is the coefficient of a, -1 is understood to be the coefficient of b, 1 is understood to be the coefficient of c **21.** polynomial, binomial
23. not a polynomial because a variable is in the denominator
25. not a polynomial because a variable is in the denominator
27. polynomial, trinomial **29.** $b - 3a$ **31.** $y + 5$
33. $x(y + z)$ **35.** $a - b$ **37.** (let x = the number) $x - 12$
39. (let x = the number) $3x + 1$ **41.** (let x = the number) $2(x + 4)$

Solutions to trial exercise problems

8. $\frac{15x^2 + y}{8}$ has one term because the fraction bar is a grouping symbol. **24.** $\frac{a+b}{5} - c$ is a polynomial. A constant can appear in the denominator, a variable cannot. **36.** $\frac{1}{2}$ of x, decreased by 2 times x would be $\frac{1}{2}x - 2x$. **39.** 3 times a number, increased by 1: If we let x represent the number, then we would have $3x + 1$.

Review exercises

1. -25 **2.** 64 **3.** -2 **4.** 15 **5.** 22 **6.** 23

Exercise 2–2

Answers to odd-numbered problems

1. 9 **3.** 5 **5.** 48 **7.** 5 **9.** 62 **11.** 288 **13.** 61 **15.** 0
17. -1 **19.** 1 **21.** -44 **23.** 20 **25.** 0 **27.** 43 **29.** $\frac{20}{3}$
31. 160 **33.** 288 **35.** 54 **37.** 2,140 **39.** 114 **41.** 256
43. 6 **45.** $\frac{15{,}000}{857}$ **47.** $\frac{540}{13}$ **49.** $\frac{400}{33}$ **51.** $85m$ **53.** $\frac{y}{10}$
55. $5n + 10d$ **57.** **a.** $p + 12$ **b.** $p - 5$ **59.** $258 - n + m$
61. $\frac{c}{50}$ **63.** $y + 2$ **65.** $12f + t$ **67.** $25{,}000n - 2{,}000$
69. $(9.95)p + (12.99)q$

Solutions to trial exercise problems

5. $(3a + 2b)(a - c) = [3(\ \ ) + 2(\ \ )][(\ \ ) - (\ \ )]$
$= [3(2) + 2(3)][(2) - (-2)] = [6 + 6][4] = [12][4] = 48$
14. $(4a + b) - (3a - b)(c + 2d) = [4(\ \ ) + (\ \ )]$
$- [3(\ \ ) - (\ \ )][(\ \ ) + 2(\ \ )] = [4(2) + (3)]$
$- [3(2) - (3)][(-2) + 2(-3)] = [8 + 3]$
$- [6 - 3][(-2) + (-6)] = [11] - [3][-8] = [11] - [-24]$
$= 35$ **31.** $I = prt$; $I = (\ \ )(\ \ )(\ \ ) = (1{,}000)(0.08)(2) = (80)(2)$
$= 160$ **39.** $A = \frac{P^2R - 120E^2}{R}$; $A = \frac{(\ \ )^2(\ \ ) - 120(\ \ )^2}{(\ \ )}$
$= \frac{(12)^2(100) - 120(5)^2}{(100)} = \frac{(144)(100) - 120(25)}{100}$
$= \frac{14{,}400 - 3{,}000}{100} = \frac{11{,}400}{100} = 114$ **47.** $V = \frac{vn}{N}$; $V = \frac{(\ \ )(\ \ )}{(\ \ )}$

$= \frac{(90)(30)}{(65)} = \frac{2{,}700}{65} = \frac{540}{13}$ **55.** n nickels is represented by $5n$ because there are 5 cents in each nickel. Therefore d dimes would be represented by $10d$. The total is represented by adding the cents from the nickels to the cents from the dimes, $5n + 10d$. **64.** If we use 11 as an example of an odd integer, the next greater odd integer would be 13. To get from 11 to 13, we must add 2. Therefore if z is an odd integer, then $z + 2$ is the next greater odd integer.

Review exercises

1. -6 **2.** -4 **3.** -8 **4.** 18 **5.** 0 **6.** -21

Exercise 2–3

Answers to odd-numbered problems

1. like **3.** like **5.** unlike **7.** $9x$ **9.** $13a + 2b$ **11.** $14x$ **13.** $-3ab$ **15.** $7x^2 + 4x$ **17.** $a^2b + 2a^3 - b^3 - 6ab^2$ **19.** $13a - 5c - 2x^2$ **21.** $-3a - 9b$ **23.** $3x^2 + 11y^2$ **25.** $3x^2 - 3x$ **27.** $x^2y^2 - 2x^2y + 5xy$ **29.** $8x^2 + 2x + 1$ **31.** $3x + 8y$ **33.** $2x + 3y$ **35.** $2a + b + 5c$ **37.** $2x + 4y$ **39.** $-3x^2y + 15xy$ **41.** $4a^3 + a^2b + 6ab^2 - 5b^3$ **43.** $70a + 9b$ **45.** $-5xy + 9y^2z + 14yz$ **47.** $-8b + 10$ **49.** $3a - 9b$ **51.** $3x^2 + 3z$ **53.** $2x$ **55.** $-4a - 3b + 4x + 2y$ **57.** $5a + 5b$ **59.** $-a - 2b$ **61.** a

Solutions to trial exercise problems

19. $3a + b + 2a - 5c - b - 2x^2 + 8a = (3a + 2a + 8a) + (b - b) - 5c - 2x^2 = (3 + 2 + 8)a + (0) - 5c - 2x^2 = 13a - 5c - 2x^2$ **45.** $(8xy + 9y^2z) - (13xy - 14yz) = 8xy + 9y^2z - 13xy + 14yz = (8xy - 13xy) + 9y^2z + 14yz = (8 - 13)xy + 9y^2z + 14yz = -5xy + 9y^2z + 14yz$ **52.** $2x - [3x - (5x - 3)] = 2x - [3x - 5x + 3] = 2x - [-2x + 3] = 2x + 2x - 3 = 4x - 3$ **59.** $-[4a + 7b - (3a + 5b)] = -[4a + 7b - 3a - 5b] = -[a + 2b] = -a - 2b$

Review exercises

1. $3x$ **2.** $6(a + 7)$ **3.** $(y - 2) \div 4$ or $\frac{y-2}{4}$

4. (let x = the number) $5x$ **5.** (let x = the number) $x - 12$

6. (let x = the number) $\frac{x}{8} - 9$

Exercise 2–4

Answers to odd-numbered problems

1. true **3.** true **5.** true **7.** false **9.** true **11.** false **13.** $\{16\}$ **15.** $\{-3\}$ **17.** $\{-2\}$ **19.** $\{-5\}$ **21.** $\{-6\}$ **23.** $\{-7\}$ **25.** $\{14\}$ **27.** $\{-1\}$ **29.** $\{4\}$ **31.** $\{-5\}$ **33.** $\{5\}$ **35.** $\{6\}$ **37.** $\{6\}$ **39.** $\{8\}$ **41.** $\{-6\}$ **43.** $\{5\}$ **45.** $\{-7\}$ **47.** $\{12\}$ **49.** $\{16\}$ **51.** $\{9\}$ **53.** 26 **55.** 33 **57.** \$735 **59.** \$365

Solutions to trial exercise problems

8. $3x + 2 = 5x - 1; \left\{\frac{3}{2}\right\}$

$$\begin{aligned} 3\left(\frac{3}{2}\right) + 2 &= 5\left(\frac{3}{2}\right) - 1 \\ \frac{9}{2} + 2 &= \frac{15}{2} - 1 \\ \frac{9}{2} + \frac{4}{2} &= \frac{15}{2} - \frac{2}{2} \\ \frac{13}{2} &= \frac{13}{2} \text{ (true)} \end{aligned}$$

23.
$$\begin{aligned} b + 7 &= 0 \\ b + 7 - 7 &= 0 - 7 \\ b &= -7 \\ &\{-7\} \end{aligned}$$
Check: $(-7) + 7 = 0$
$0 = 0$ (true)

28.
$$\begin{aligned} -y - 6 &= -2y + 1 \\ -y + 2y - 6 &= -2y + 2y + 1 \\ y - 6 &= 1 \\ y - 6 + 6 &= 1 + 6 \\ y &= 7 \\ &\{7\} \end{aligned}$$
Check: $-(7) - 6 = -2(7) + 1$
$-13 = -14 + 1$
$-13 = -13$ (true)

38.
$$\begin{aligned} 5(x + 2) &= 4(x - 1) \\ 5x + 10 &= 4x - 4 \\ 5x - 4x + 10 &= 4x - 4x - 4 \\ x + 10 &= -4 \\ x + 10 - 10 &= -4 - 10 \\ x &= -14 \\ &\{-14\} \end{aligned}$$

45.
$$\begin{aligned} 3(z + 7) - (8 + 2z) &= 6 \\ 3z + 21 - 8 - 2z &= 6 \\ z + 13 &= 6 \\ z + 13 - 13 &= 6 - 13 \\ z &= -7 \\ &\{-7\} \end{aligned}$$

58. Let b = the original balance.

original balance	makes a deposit	of \$42.50	equals	new balance
b	$+$	42.50	$=$	125.30

$b + 42.50 = 125.30$
$b + 42.50 - 42.50 = 125.30 - 42.50$ Subtract 42.50 from both members.
$b = 82.80$
The original balance was \$82.80.

Review exercises

1. 16 **2.** -12 **3.** 1 **4.** 1 **5.** 1 **6.** 1

Exercise 2–5

Answers to odd-numbered problems

1. $\{4\}$ **3.** $\{6\}$ **5.** $\{16\}$ **7.** $\{35\}$ **9.** $\{12\}$ **11.** $\{-3\}$ **13.** $\{-4\}$ **15.** $\{7\}$ **17.** $\{-4\}$ **19.** $\left\{\frac{7}{3}\right\}$ **21.** $\left\{\frac{3}{2}\right\}$ **23.** $\{0\}$ **25.** $\{0\}$ **27.** $\{15\}$ **29.** $\{-14\}$ **31.** $\{4\}$ **33.** $\{-7\}$ **35.** $\{11\}$ **37.** $\{-26\}$ **39.** $\left\{\frac{112}{3}\right\}$ **41.** 9 **43.** -63 **45.** \$4.50 **47.** \$130 **49.** 64

Solutions to trial exercise problems

7.
$$\begin{aligned} \frac{1}{7}x &= 5 \\ 7 \cdot \frac{1}{7}x &= 7 \cdot 5 \\ x &= 35 \\ &\{35\} \end{aligned}$$
Check: $\frac{1}{7}(35) = 5$
$5 = 5$ (true)

15.
$$\begin{aligned} -4x &= -28 \\ \frac{-4x}{-4} &= \frac{-28}{-4} \\ x &= 7 \\ &\{7\} \end{aligned}$$
Check: $-4(7) = -28$
$-28 = -28$ (true)

23. $5x = 0$
$$\frac{5x}{5} = \frac{0}{5}$$
$x = 0$
$\{0\}$
Check: $5(0) = 0$
$0 = 0$ (true)

27.
$$\frac{x}{3} = 5$$
$$3 \cdot \frac{x}{3} = 3 \cdot 5$$
$x = 15$
$\{15\}$
Check: $\frac{(15)}{3} = 5$
$5 = 5$ (true)

31. $2.6x = 10.4$
$$\frac{2.6x}{2.6} = \frac{10.4}{2.6}$$
$x = 4$
$\{4\}$
Check: $2.6(4) = 10.4$
$10.4 = 10.4$ (true)

38.
$$\frac{5}{7}x = 8$$
$$\frac{7}{5} \cdot \frac{5}{7}x = \frac{7}{5} \cdot 8$$
$$x = \frac{56}{5}$$
$$\left\{\frac{56}{5}\right\}$$
Check: $\frac{5}{7}\left(\frac{56}{5}\right) = 8$
$8 = 8$ (true)

45. Let w = Nancy's hourly wage.

30 hours	at	hourly wage	is	\$135.00
30	$\cdot$	w	$=$	135

$30w = 135$
$$\frac{30w}{30} = \frac{135}{30} \quad \text{Divide both members by 30.}$$
$w = 4.5$
Nancy's hourly wage is \$4.50.

Review exercises

1. $5x - 2$ 2. $2x + 1$ 3. $12x + 2$
4. $3x - 5$ 5. $10x - 1$ 6. $5x + 1$

Exercise 2-6

Answers to odd-numbered problems

1. $\{2\}$ 3. $\{-2\}$ 5. $\{36\}$ 7. $\left\{\frac{16}{3}\right\}$ 9. $\{4\}$ 11. $\{0\}$

13. $\{3\}$ 15. $\{0\}$ 17. $\{1\}$ 19. $\left\{\frac{8}{5}\right\}$ 21. $\{-10\}$

23. $\left\{-\frac{9}{2}\right\}$ 25. $\{18\}$ 27. $\left\{-\frac{51}{8}\right\}$ 29. $\{12\}$ 31. $\{3\}$

33. $\left\{\frac{10}{7}\right\}$ 35. $\left\{\frac{16}{11}\right\}$ 37. $\left\{\frac{37}{10}\right\}$ 39. $\left\{\frac{2}{5}\right\}$ 41. $\{3\}$

43. $\{1\}$ 45. a. $-\frac{70}{9}°\ C$ or $-7\frac{7}{9}°\ C$ b. $-\frac{295}{9}°\ C$ or $-32\frac{7}{9}°\ C$ c. $-\frac{50}{3}°\ C$ or $-16\frac{2}{3}°\ C$ 47. $\frac{7}{3}$

Solutions to trial exercise problems

31.
$$\begin{aligned} 3(2x - 1) &= 4x + 3 \\ 6x - 3 &= 4x + 3 \\ 6x - 4x - 3 &= 4x - 4x + 3 \\ 2x - 3 &= 3 \\ 2x - 3 + 3 &= 3 + 3 \\ 2x &= 6 \\ \frac{2x}{2} &= \frac{6}{2} \\ x &= 3 \end{aligned}$$
$\{3\}$

35.
$$\begin{aligned} 8 - 2(3x + 4) &= 5x - 16 \\ 8 - 6x - 8 &= 5x - 16 \\ -6x &= 5x - 16 \\ -6x + 6x &= 5x + 6x - 16 \\ 0 &= 11x - 16 \\ 0 + 16 &= 11x - 16 + 16 \\ 16 &= 11x \\ \frac{16}{11} &= \frac{11x}{11} \\ \frac{16}{11} &= x \end{aligned}$$
$$\left\{\frac{16}{11}\right\}$$

46b.
$$\begin{aligned} W &= 243,\ T = -3 \\ W &= KT^4 \\ (243) &= K(-3)^4 \\ 243 &= K \cdot 81 \\ \frac{243}{81} &= \frac{K \cdot 81}{81} \\ 3 &= K \end{aligned}$$
Hence the value of K is 3.

Review exercises

1. 108 2. 144 3. 88 4. 360 5. 280 6. 92

Exercise 2-7

Answers to odd-numbered problems

1. $w = \frac{V}{\ell h}$ 3. $P = \frac{I}{rt}$ 5. $m = \frac{F}{a}$ 7. $V = \frac{K}{P}$

9. $R = \frac{W}{I^2}$ 11. $w = \frac{P - 2\ell}{2}$ 13. $a = P - b - c$

15. $a = \frac{by + c + 3}{y}$ 17. $k = V - gt$ 19. $b = \frac{2A - ch}{h}$

21. $a = \ell - dn + d$ 23. $P = \frac{A}{1 + r}$ 25. $f = \frac{T - g}{2}$

27. $q = \frac{D - R}{d}$ 29. $c = \frac{W - b^2 - R}{2b}$ 31. $r = \frac{A - P}{Pt}$

33. $a = \frac{V + br^2}{r^2}$ 35. $x = -6y$ 37. $g = \frac{2vt - 2S}{t^2}$

39. $g = \frac{2s - 2vt}{t^2}$ 41. $S = \frac{P + Cn + e}{n}$

43. $e = nS - Cn - P$

Solutions to trial exercise problems

18. $V = k + gt$, for t
$$\begin{aligned} V &= k + gt \\ V - k &= gt \\ \frac{V - k}{g} &= t \end{aligned}$$

22. $\ell = a + (n - 1)d$, for d
$$\begin{aligned} \ell &= a + (n - 1)d \\ \ell - a &= (n - 1)d \\ \frac{\ell - a}{n - 1} &= d \end{aligned}$$

29. $R = W - b(2c + b)$, for c
$$\begin{aligned} R &= W - 2bc - b^2 \\ R + 2bc &= W - b^2 \\ 2bc &= W - b^2 - R \\ c &= \frac{W - b^2 - R}{2b} \end{aligned}$$

Review exercises

1. -25 **2.** 25 **3.** -81 **4.** -27 **5.** x^4
6. (let x = the number) x^2 **7.** ab **8.** xy

Exercise 2–8

Answers to odd-numbered problems

1. 22, 40 **3.** 35, 52 **5.** 28 **7.** 7 **9.** 54 **11.** 21 **13.** 15 **15.** 29, 34 **17.** 12, 108 **19.** 15, 17, 19 **21.** 12, 84 **23.** 20, 22, 24 **25.** 30, 31, 32 **27.** 14, 7, 42 **29.** 5, 9 **31.** 24 **33.** 9, 10, 11 **35.** 34, 36, 38 **37.** 19 feet, 16 feet **39.** 9 inches, 26 inches **41.** 18 feet, 23 feet **43.** 4 feet, 8 feet **45.** 19 feet, 31 feet **47.** 22 teeth, 37 teeth **49.** $28\frac{1}{2}$ volts, $60\frac{1}{2}$ volts
51. \$13,000 at 8%; \$7,000 at 6% **53.** \$11,000 at 8%; \$4,000 at 6% **55.** \$8,000 at 10%; \$10,000 at 8% **57.** \$8,000 at 9%; \$22,000 at 7% **59.** \$8,000 at 11%; \$17,000 at 18% **61.** \$7,700 at 13%; \$13,300 at 9% **63.** \$5,000 **65.** \$14,000

Solutions to trial exercise problems

5. number equation

$x \qquad \frac{x}{4} + 6 = 13$

Solution: $\frac{x}{4} + 6 = 13$

$$\frac{x}{4} = 7$$
$$x = 28$$

26. first second third equation

$x \qquad 3 \cdot x \qquad x - 6 \qquad (x) + (3 \cdot x) + (x - 6) = 44$

Solution: $(x) + (3x) + (x - 6) = 44$

$$x + 3x + x - 6 = 44$$
$$5x - 6 = 44$$
$$5x = 50$$
$$x = 10$$

First is 10, second ($3x$) is $3(10) = 30$, and the third ($x - 6$) is $(10) - 6 = 4$.

43. shorter piece longer piece equation

$x \qquad x + 4 \qquad x + (x + 4) = 12$

Solution: $x + (x + 4) = 12$

$$x + x + 4 = 12$$
$$2x + 4 = 12$$
$$2x = 8$$
$$x = 4$$

Shorter piece is 4 feet and the longer piece ($x + 4$) is $(4) + 4 = 8$, 8 feet.

59. number of dollars at 11% loss; number of dollars at 18% profit; equation

$x \qquad 25{,}000 - x \qquad (25{,}000 - x)(0.18) - x(0.11) = 2{,}180$

Solution: $(25{,}000 - x)(0.18) - x(0.11) = 2{,}180$

$$(25{,}000)(0.18) - (0.18)x - (0.11)x = 2{,}180$$
$$4{,}500 - (0.29)x = 2{,}180$$
$$4{,}500 = (0.29)x + 2{,}180$$
$$2{,}320 = (0.29)x$$
$$8{,}000 = x$$

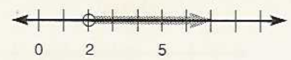
Therefore \$8,000 was invested at the 11% loss and \$25,000 − 8,000 = \$17,000 was invested at 18% profit.

Review exercises

1. 100 **2.** 84 **3.** 204 **4.** 108 **5.** 3,180 **6.** 16 **7.** 256 **8.** 40

Exercise 2–9

Answers to odd-numbered problems

1.


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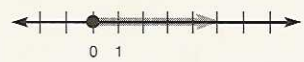

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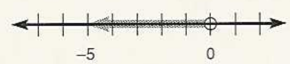

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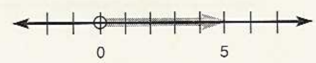
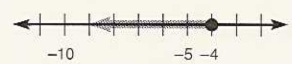
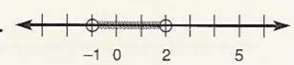
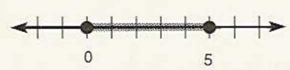
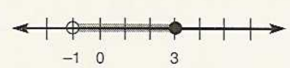
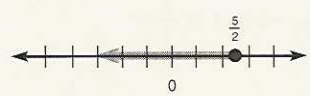


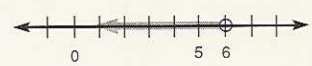
9.


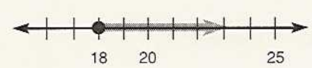
11.


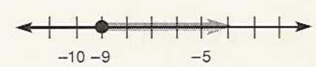
13.


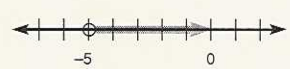
15.


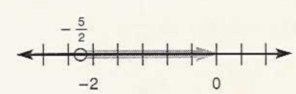
17. $x \leq \frac{5}{2}$;


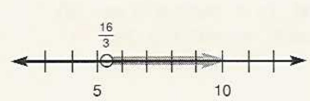
19. $x < 6$;


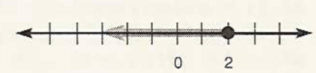
21. $x \geq 18$;


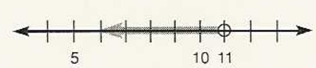
23. $x \geq$


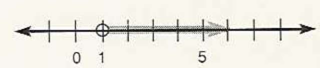
25. $x > -5$;


27. $x >$


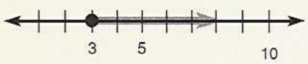
29. $x > \frac{16}{3}$;


31. $x \leq 2$;


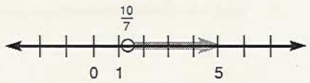
33. $x < 11$;


35. $x > 1$;


37. $x \geq 3$;

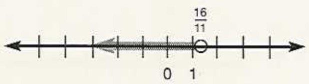


39. $x > \frac{10}{7}$;

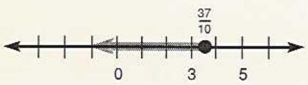


41. $x > -10$;

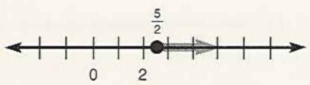
43. $x < \frac{16}{11}$;



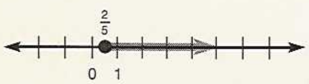
45. $x \leq \frac{37}{10}$;



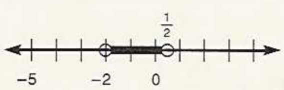
47. $x \geq \frac{5}{2}$;



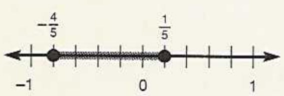
49. $x \geq \frac{2}{5}$;



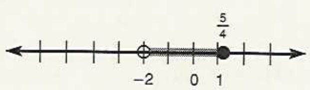
51. $-2 < x < \frac{1}{2}$;



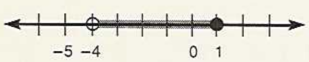
53. $-\frac{4}{5} \leq x \leq \frac{1}{5}$;



55. $-2 < x \leq \frac{5}{4}$;

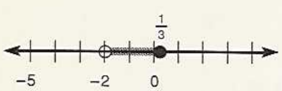


57. $-4 < x \leq 1$;



59. $-1 \leq x < 7$;

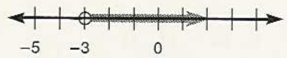
61. $-2 < x \leq \frac{1}{3}$;



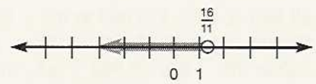
63. (x = student's score) $x \geq 72$ **65.** (x = number of new lift trucks) $x \geq 8$ **67.** (P = selling price, C = cost) $P \geq 2C$
69. $x < 6$ **71.** $x \geq 12$ **73.** $x \geq 7$ **75.** $1 < x < 5$
77. $5 < x < 27$ **79.** $x > 18$

Solutions to trial exercise problems

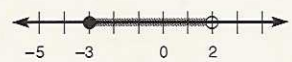
22.
$$-4x < 12$$
$$\frac{-4x}{-4} > \frac{12}{-4}$$
$$x > -3$$



43.
$$8 - 2(3x + 4) > 5x - 16$$
$$8 - 6x - 8 > 5x - 16$$
$$-6x > 5x - 16$$
$$-6x + 6x > 5x + 6x - 16$$
$$0 > 11x - 16$$
$$0 + 16 > 11x - 16 + 16$$
$$16 > 11x$$
$$\frac{16}{11} > \frac{11x}{11}$$
$$\frac{16}{11} > x$$
$$x < \frac{16}{11}$$

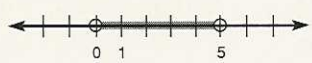


56.
$$-2 < -x \leq 3$$
$$-2 < -1 \cdot x \leq 3$$
$$\frac{-2}{-1} > \frac{-1 \cdot x}{-1} \geq \frac{3}{-1}$$
$$2 > x \geq -3$$
$$-3 \leq x < 2$$



66. A company will hire at least 2 new employees, but not more than 7. Let x be the number of new employees. Then the inequality would be $2 \leq x \leq 7$.

75. Two times a number plus 4 is greater than 6 but less than 14. Let x be the number. Then the inequality would be $6 < 2x + 4 < 14$.
Solving: $6 - 4 < 2x + 4 - 4 < 14 - 4$
$$2 < 2x < 10$$
$$\frac{2}{2} < \frac{2x}{2} < \frac{10}{2}$$
$$1 < x < 5$$



78. The perimeter of a rectangle must be less than 100 feet. If the length is known to be 30 feet, find all numbers that the width could be. Let x represent the width of the rectangle. Then the inequality would be $60 < 2 \cdot 30 + 2 \cdot x < 100$. Since the width of a real rectangle must be greater than zero and we already know the length to be 30, then the smallest value for the perimeter must be greater than 60 (two times the known length).
Solving:
$$60 < 2 \cdot 30 + 2x < 100$$
$$60 < 60 + 2x < 100$$
$$60 - 60 < 60 - 60 + 2x < 100 - 60$$
$$0 < 2x < 40$$
$$\frac{0}{2} < \frac{2x}{2} < \frac{40}{2}$$
$$0 < x < 20$$
The width of the rectangle must be some real number greater than zero feet but less than 20 feet.

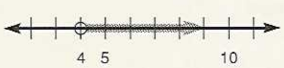
Review exercises

1. -16 **2.** 16 **3.** -16 **4.** 16 **5.** x^5 **6.** (let x = the number) x^3 **7.** (let x = the number) x^2 **8.** xy

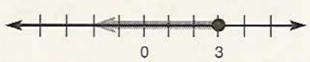
Chapter 2 review

1. 3 **2.** 1 **3.** 2 **4.** 2 **5.** polynomial **6.** polynomial **7.** polynomial **8.** not a polynomial because a variable is in the denominator **9.** $5x$ **10.** $y - 7$ **11.** $z + 4$ **12.** (let x = the number) $2x + 6$ **13.** 1 **14.** -1 **15.** 72 **16.** -4 **17.** 4 **18.** 7 **19. a.** 3 **b.** $\frac{189}{4}$ **20.** 1,040 **21.** $4x^2 - 3x + 3$ **22.** $-a^2 + a + 11$ **23.** $-6a^2$ **24.** $5x^3 - 7xy^2 - 2y^3 - 4x^2$ **25.** $-11ab + 7b^2c + 11bc$ **26.** $-6y + 1$ **27.** $5ab + 3ac - 4bc$ **28.** -5 **29.** $7x + y$ **30.** $2x + 5y$ **31.** $-6x + 4y$ **32.** $9a - 10b$ **33.** true **34.** false **35.** false **36.** true **37.** $\{7\}$ **38.** $\{21\}$ **39.** $\{-11\}$ **40.** $\{-6\}$ **41.** $\{4\}$ **42.** $\{19\}$ **43.** $\{-2\}$ **44.** $\{9\}$ **45.** $\{3\}$ **46.** $\{3\}$ **47.** $\{-7\}$ **48.** $\{-7\}$ **49.** $\{12\}$ **50.** $\{14\}$ **51.** $\{15\}$ **52.** $\{21\}$ **53.** $\left\{\frac{21}{4}\right\}$ **54.** $\{-18\}$ **55.** $\left\{-\frac{9}{2}\right\}$ **56.** $\{35\}$ **57.** $\{0\}$ **58.** $\{4\}$ **59.** $\{6\}$ **60.** $\{-8\}$ **61.** $\{3\}$ **62.** $\left\{\frac{14}{3}\right\}$ **63.** $\left\{\frac{7}{3}\right\}$ **64.** $\{3\}$ **65.** $\left\{\frac{1}{2}\right\}$ **66.** $\left\{-\frac{22}{3}\right\}$ **67.** $\left\{-\frac{1}{4}\right\}$ **68.** $\left\{\frac{7}{13}\right\}$ **69.** $\left\{-\frac{5}{2}\right\}$ **70.** $\{-14\}$ **71.** $\{-24\}$ **72.** $\left\{\frac{7}{4}\right\}$ **73.** $\{0\}$ **74.** $\left\{\frac{1}{5}\right\}$ **75.** $\left\{-\frac{7}{4}\right\}$ **76.** $\{2\}$ **77.** $a = \frac{F}{m}$ **78.** $I = \frac{E}{R}$ **79.** $P = \frac{k}{V}$ **80.** $g = V - k - t$ **81.** $c = \frac{2A - bh}{h}$ **82.** $x = \frac{4y}{3}$ **83.** 64 and 41 **84.** 36 **85.** 45 **86.** \$11,000 at 8%; \$9,000 at 7% **87.** \$12,000 at 12%; \$13,000 at 19%

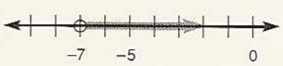
88. $x > 4$;



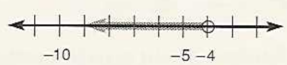
89. $x \leq 3$;



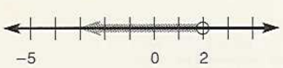
90. $x > -7$;



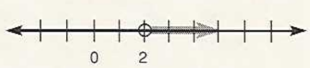
91. $x < -4$;



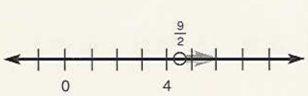
92. $x < 2$;



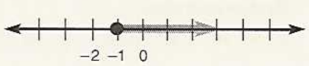
93. $x > \frac{15}{7}$;



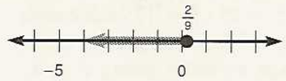
94. $x > \frac{9}{2}$;



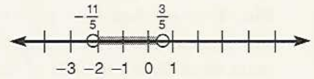
95. $x \geq -\frac{6}{5}$;



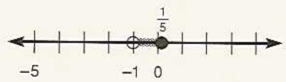
96. $x \leq \frac{2}{9}$;



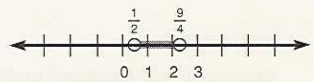
97. $-\frac{11}{5} < x < \frac{3}{5}$;



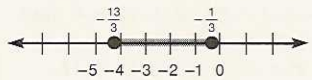
98. $-1 < x \leq \frac{1}{5}$;



99. $\frac{1}{2} < x < \frac{9}{4}$;



100. $-\frac{13}{3} \leq x \leq -\frac{1}{3}$;



Chapter 2 cumulative test

1. -12 **2.** 4 **3.** 3 **4.** undefined **5.** -25 **6.** $\frac{1}{2}$ **7.** $-\frac{1}{6}$ **8.** 0 **9.** 19.78 **10.** 41 **11.** 38 **12.** -24 **13.** 26 **14.** 6 **15.** 20 **16.** $4x$ **17.** $2x^2y^2 + 3xy$ **18.** $a + 3b$ **19.** $4x^2y - 7xy^2$ **20.** $a^3 + 2a^2 + a - 1$ **21.** $-3x + 4y$ **22.** $4a + 2b$ **23.** 64 **24.** -25 **25.** 234 **26.** 2 **27.** 4 **28.** 42 **29.** 51 mph **30.** $x - y$ **31.** (let x = the number) $x + 6$ **32.** $10d + 5n + c$ **33.** $\left\{\frac{5}{3}\right\}$ **34.** $\{0\}$ **35.** $\left\{-\frac{19}{2}\right\}$ **36.** $\left\{\frac{17}{16}\right\}$ **37.** $\left\{\frac{30}{11}\right\}$ **38.** $x \leq -6$ **39.** $x < -7$ **40.** $x > \frac{8}{5}$ **41.** $-2 < x < 4$ **42.** $-1 \leq x \leq 6$ **43.** $b = P - a - c$ **44.** $y = \frac{x - az}{a}$ **45.** \$6,000 at 6%; \$4,000 at 5% **46.** 14,16,18 **47.** 12 **48.** \$6,500 at 12% profit; \$10,500 at 19% loss **49.** $x \leq 6$

Chapter 3

Exercise 3–1

Answers to odd-numbered problems

1. a^5 **3.** $(-2)^4$ **5.** x^6 **7.** $(xy)^4$ **9.** $(x - y)^3$ **11.** $xxxx$ **13.** $(-2)(-2)(-2)$ **15.** $5 \cdot 5 \cdot 5$ **17.** $(4y)(4y)(4y)(4y)$ **19.** $(x - y)(x - y)$ **21.** x^{11} **23.** R^3 **25.** a^9 **27.** $5^5 = 3{,}125$ **29.** $4^7 = 16{,}384$ **31.** $(x - 2y)^{10}$ **33.** $(a - b)^{11}$ **35.** x^4y^4 **37.** $64x^3y^3z^3$ **39.** x^{15} **41.** b^{25} **43.** $6x^4y^3$ **45.** a^7b^5 **47.** $30x^5$ **49.** $12a^7b^7c$ **51.** $12a^5b^3$ **53.** $-6a^3b^5$ **55. a.** $V = 5^3$, 125 cubic units **b.** $V = 4^3$, 64 cubic units **c.** $V = 6^3$, 216 cubic units **57.** $s = \frac{1}{2}gt^2$ **59.** $V = \frac{4}{3}\pi r^3$ **61.** $m^4 - 8$ **63.** $2x^2 - y^3$ **65.** $\frac{p^3}{q^2}$

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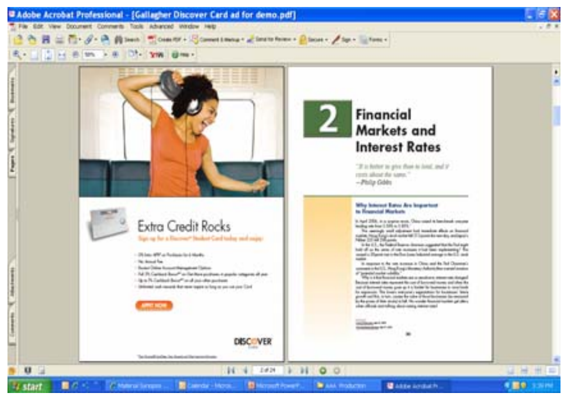

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